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Pobočka Košice

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Ústav matematiky

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Katedra aplikovanej matematiky a informatiky

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# **23. Konferencia**

# **košických matematikov**

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**Herľany**  
**27. – 29. marca 2025**



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VEDECKOTECHNICKÝCH  
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## Predhovor

Vážení účastníci Konferencie košických matematikov a ostatní čitatelia!

Práve otvárate Zborník abstraktov príspevkov z Konferencie košických matematikov, ktorá v roku 2025 píše už 23. kapitolu svojej histórie. Táto je neodmysliteľne vyše štvrtstoročia spätá s jarnými mesiacmi a Učebno-výcvikovým zariadením Technickej univerzity v Košiciach v Herľanoch. Aj keď sa snažíme o každoročné konanie podujatia, nevšedné udalosti niektorých rokov zanechali nejaké diery na línii pravidelnosti usporiadania série týchto stretnutí priaznivcov matematiky.

Konferencia vyniká svojou príjemnou rodinnou atmosférou, ktorú, ako dúfame, oceníte aj vy. Vďaka nej môžeme lepšie reagovať na požiadavky účastníkov a podľa možností im vyhovieť. Práve na podnet tých minulo-ročných sa po desiatich rokoch opäť vraciame k marcovému dátumu jej konania. Aj keď sa ozývali hlasy i na zmenu rámca jej trvania, rozhodli sme sa zachovať trojdňový formát. Tento je totiž aj z hľadiska intervalu erupcie unikátneho studenododného gejzíru v Herľanoch optimálny. Počas takto dlhého trvania konferencie je aspoň jedna jeho erupcia, takpovediac, zaručená.

Odborný obsah podujatia je zabezpečený sériou pozvaných prednášok i prihlásených referátov účastníkov. Pozvanie prednášať na 23. Konferencii košických matematikov prijali:

- Mgr. Anino Belan, PhD. (Škola pre mimoriadne nadané deti a Gymnázium v Bratislave),
- doc. Mgr. Ján Brajerčík, Ph.D. (Prešovská univerzita v Prešove),
- doc. RNDr. Martina Hančová, PhD. (Univerzita Pavla Jozefa Šafárika v Košiciach),
- RNDr. Dag Hrubý (Univerzita Palackého v Olomouci),
- Mgr. Samuel Kováčik, PhD. (Univerzita Komenského v Bratislave),
- prof. RNDr. Lubomír Snoha, DSc., DrSc. (Univerzita Mateja Bela v Banskej Bystrici).

Veríme, že svojimi prednáškami zaujmú široké publikum. Ich abstrakty, ako aj abstrakty prihlásených príspevkov spolu s programom podujatia a kontaktami na jeho účastníkov nájdete v tejto publikácii. Pozitívny ohlas z minulých rokov nás motivoval do programu opätovne začleniť i didaktický blok prednášok. Sme radi, že čoraz viac pedagógov aj z radov SŠ a ZŠ si

vo svojom programe na ne nájde čas. Srdečne vás pozývame predostrieť svoje problémy a hľadať a nachádzať ich riešenia spolu s ostatnými účastníkmi konferencie v diskusiách okrúhleho stola.

23. Konferencia košických matematikov sa koná pod záštitou Jednoty slovenských matematikov a fyzikov pri SAV – pobočka Košice, v spolupráci s Ústavom riadenia a informatizácie výrobných procesov FBERG TUKE, Katedrou aplikovanej matematiky a informatiky Sjf TUKE, Ústavom matematických vied Prírodovedeckej fakulty UPJŠ a pobočkou Slovenskej spoločnosti aplikovanej kybernetiky a informatiky pri ÚRIVP FBERG TUKE – členom Zväzu slovenských vedeckotechnických spoločností. V mene organizátorov dúfame, že si z konferencie odnesiete len pozitívne dojmy.

Editori: Ján Buša  
Erika Fecková Škrabuláková  
Andrea Feňovčíková

## Editorial

Dear participants of the Conference of Košice Mathematicians and other readers!

You are now opening the booklet of abstracts of contributions presented at the Conference of Košice Mathematicians, which in 2025 writes the 23<sup>rd</sup> chapter of its history. This event has been inherently connected to the spring months and the Education Training Facility of the Technical University of Košice in Herľany for over a quarter of a century. Although we strive to hold the event annually, extraordinary events in some years have left gaps in the regularity of these mathematical meetings.

The conference is distinguished by its pleasant family atmosphere, which we hope you will appreciate as well. Thanks to this, we can better respond to participants' requests and meet them whenever possible. At the suggestion of last year's participants, we are returning to a March date for the conference after ten years. Although there were voices calling for a change in the duration of the event, we decided to maintain the three-day format. This is, in fact, optimal in terms of the eruption interval of the unique cold-water geyser in Herľany. During this extended duration, at least one eruption is, so to speak, guaranteed.

The scientific content of the event is ensured by a series of invited lectures and the contributions submitted by other participants. The following speakers have accepted the invitation to give a lecture at the 23<sup>rd</sup> Conference of Košice Mathematicians:

- Mgr. Anino Belan, PhD. (School for Gited Children and Grammar School in Bratislava),
- doc. Mgr. Ján Brajerčík, Ph.D. (University of Prešov),
- doc. RNDr. Martina Hančová, PhD. (Pavol Jozef Šafárik University in Košice),
- RNDr. Dag Hrubý (Palacký University Olomouc),
- Mgr. Samuel Kováčik, PhD. (Comenius University Bratislava),
- prof. RNDr. Lubomír Snoha, DSc., DrSc. (Matej Bel University in Banská Bystrica).

We believe that their lectures will captivate a wide audience. Their abstracts, as well as the abstracts of the submitted contributions, the program of the event, and contact information for the participants, can be found

in this publication. The positive feedback from previous years has motivated us to once again include a didactic block of lectures in the program. We are pleased that more and more teachers from both secondary and primary schools are finding time for these sessions in their schedules, as well. We warmly invite you to present your problems and seek and find their solutions together with the other conference participants in the round-table discussions.

The 23<sup>rd</sup> Conference of Košice Mathematicians is held under the auspices of the Union of Slovak Mathematicians and Physicists by Slovak Academy of Science – Košice Branch, in cooperation with the Department of Control and Informatization of Production Processes of FBERG TUKE, the Department of Applied Mathematics and Informatics at the Faculty of Engineering of TUKE, the Institute of Mathematical Sciences at the Faculty of Science of UPJŠ, and the branch of the Slovak Society for Applied Cybernetics and Informatics at ÚRIVP FBERG TUKE – a member of the Association of Slovak Scientific and Technological Societies. On behalf of the organizers, we hope that you will take away only positive impressions from the conference.

Editors: Ján Buša  
Erika Fecková Škrabuláková  
Andrea Feňovčíková

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# Invited lectures

## Very old mathematics

**Anino Belan**

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In this lecture we will try to read and interpret the Old Babylonian clay tablets YBC 7289 and Plimpton 322. Along with this we will mention several interesting achievements made by Babylonian mathematics before 1750 BC.

**Keywords.** History of mathematics, Babylon, clay tablets, cuneiform script.

## Global variational geometry: structures, methods, problems

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The global variational geometry is a branch of mathematics devoted to extremal problems on the frontiers of differential geometry, topology, global analysis, algebra, calculus of variations and mathematical physics. It generalizes classical calculus of variations, where underlying Euclidean spaces are replaced by smooth manifolds and fibred spaces, and Lagrange functions are replaced by Lagrange differential forms. The subject of global variational geometry is to study extremals of integral variational functionals for sections of fibred manifolds, corresponding differential equations, and objects invariant under transformations of underlying geometric structures.

In the contribution we introduce basic concepts and structures concerning global variational geometry such as topological, smooth and fibred manifolds, a jet, a differential form, a Lagrangian, a variational structure, the Euler-Lagrange equations. Some examples of problems solved by methods of global variational geometry, namely the Hilbert variational problem, the inverse problem of the calculus of variations and the Zermelo navigation problem, are also discussed.

**Keywords.** Global variational geometry, fibred manifold, jet, Lagrangian, Euler-Lagrange equations.

**Acknowledgement.** The present work was supported by the grant KEGA 021PU-4/2024.

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# Integral transforms in statistics, measurements and data science

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In metrology, traditional methods often struggle with current measurement challenges, e.g., complex scenarios with a linear combination of random variables (RVs) representing various (in)dependent uncertainty sources, including non-Gaussian distributions. In fields such as medicine, social sciences, and education, effect sizes typically assessed through ratio statistics are essential for evaluating pre- and post-intervention outcomes, or differences between groups.

From a mathematical statistics viewpoint, all these cases require handling algebraic operations  $+$ ,  $-$ ,  $\times$ ,  $/$  on (in)dependent RVs, which will be addressed in the first part. Here we also highlight the Mellin integral transform, a powerful tool for analyzing products, ratios, and more general algebraic functions of independent RVs. Its importance parallels the Fourier transform, associated with the characteristic function, which is effectively utilized for sums and differences.

The second part focuses on the practical implementation of algebraic operations on RVs in a fast, reliable, and accurate manner. A numerical approach emerges as a universally applicable solution. Using realistic contexts, we demonstrate numerical quadrature methods, specifically the simple trapezoidal rule and its sophisticated optimal version, the double exponential quadrature. Additionally, we show the strength of open data science in uniting diverse open computational tools (Jupyter, Scientific Python, SageMath, C/C++ libraries) into a coherent framework, ensuring availability, transparency, comprehensibility, and customization for numerical analysis and computations.

**Keywords.** Algebra of random variables, Fourier transform, Mellin transform, numerical quadrature, double exponential quadrature, open data science tools.

**Acknowledgement.** This work was supported by the Slovak Research and Development Agency under the contract No. APVV-21-0216, APVV-21-0369, and by the Slovak Scientific Grant Agency VEGA under grant VEGA 1/0585/24.

## Scholé

Dag Hrubý

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The lecture will begin with a commemoration of the 800<sup>th</sup> anniversary of the famous mathematical competition at the court of Emperor Frederick II in Pisa in 1225. Leonardo of Pisa, known as Fibonacci, was introduced to the Emperor. Three problems given to Leonardo by John of Palermo [1], the emperor's mathematician, as part of this competition will be shown.

The main aim of the lecture is to show how the function of school and education has changed from antiquity to the present day. The beginnings of the development of European thought and education will be recalled. Attention will be paid to the role of mathematics in this development.

The conclusion of the lecture will be devoted to some of the risks posed by the neo-liberal approach to education. The main inspiration came from texts [2], [3] and [4]. The problem of the meaning of education in the present time will be shown. It seems to be a problem to define a clear aim of education and training at the present time. Should more emphasis be placed on training specialists in various fields of science and technology, or should more attention be paid to what can be described as care for the soul, emphasis on tradition, on proven and shared knowledge, attitudes and values? It seems that the meaning of education is no longer clear not only to students but also to teachers.

**Keywords.** Scholé, teacher, education, educational goal.

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## The fuzzy onion

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We present a matrix model realization of a three-dimensional quantum space, featuring an onion-like structure composed of concentric fuzzy spheres with increasing radii [1]. The angular component of the Laplace operator is derived from that of the fuzzy sphere, while the radial component is constructed using operators that connect matrices of different sizes through the matrix harmonic expansion. This framework will be introduced in a pedagogical manner, making it accessible to a broad audience, including those unfamiliar with matrix models. Key concepts and methods will be explained step by step, with illustrative examples to aid understanding. As an illustration of this approach, we conduct a numerical simulation of a scalar quantum field theory, examine classical heat transfer, analyze the quantum mechanical hydrogen atom, and explore certain analytical aspects of the scalar field theory in this space.

It is known that fields on a single (fuzzy) sphere can be encoded in a Hermitian matrix. We proposed functions of three dimensions to be encoded in a matrix of a specific form

$$\Psi = \begin{pmatrix} \Phi^{(1)} & & & \\ & \Phi^{(2)} & & \\ & & \ddots & \\ & & & \Phi^{(M)} \end{pmatrix}.$$

The fields across various layers are connected by performing a Fourier transformation followed by adding or discharging unmapable degrees of

freedom. As a result, we construct a model of three-dimensional quantum space with spherical symmetry.

**Keywords.** Matrix models, fuzzy space.

**Acknowledgement.** This research was supported by VEGA 1/0025/23 grant Matrix models and quantum gravity and MUNI Award for Science and Humanities funded by the Grant Agency of Masaryk University.

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<http://dx.doi.org/10.1103/PhysRevD.109.105004>

# Dynamical systems: Connecting a math olympiad problem and Benford's law

Lubomír Snoha

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A dynamical system  $(X, f)$ , given by a metric space  $X$  and a continuous map  $f: X \rightarrow X$ , is called minimal if the orbit  $\{x, f(x), f(f(x)), f(f(f(x))), \dots\}$  of every point  $x \in X$  is dense in  $X$ . Minimal dynamical systems are, in a sense, the most fundamental ones, as they have no nontrivial (closed) subsystems. They belong to the favorite topics of investigation for the lecturer, whose main contributions relate to the properties of minimal maps and the topological characterization of minimal sets (see, e.g., [1, 2, 3, 4, 5, 6]). A substantial part of this talk is based on [7, 8] and can be viewed as a motivation for the study of minimal dynamical systems.

We prove that the power  $2^n$  can have any first digit (the so-called Gelfand's problem) and, more generally, that for every group of digits in which the first digit is not zero, there are infinitely many positive integers  $n$  such that the decimal representation of  $2^n$  starts with the given group of digits (this is the Mathematical Olympiad problem mentioned in the title of the talk). We will demonstrate that the true reason these claims hold is that the irrational rotation of the circle is a minimal dynamical system. This fact appeared in principle as early as the 14<sup>th</sup> century in Oresme and was first proved rigorously by Kronecker in 1884.

By using deeper properties of the irrational rotation, it is even possible to find the frequencies of the first digits in the Gelfand’s problem and the frequencies of the initial blocks in the Olympiad problem. Additionally, there are many other sequences for which those frequencies are the same as for the sequence of powers of two; these are called Benford sequences.

In particular, for the sequence  $2^n$  and, more generally, for all Benford sequences, the frequencies of first digits are given by  $\log_{10}(1 + 1/d)$ , where  $d = 1, 2, \dots, 9$ . Thus, the digit 1 appears as the first digit about 30 % of the time, while larger digits occur less frequently, with the digit 9 appearing as the first digit only about 5 % of the time. Surprisingly, approximately the same frequencies  $\log_{10}(1 + 1/d)$  for  $d = 1, 2, \dots, 9$  appear as the frequencies of first digits in many (though not all) real-world numerical datasets. This statistical principle, due to Newcomb (1881) and Benford (1938), is called Benford’s Law of First Digits. Benford’s law is applicable in various fields, including finance, accounting, and fraud detection, as deviations from the expected distribution of first digits may indicate anomalies or potential data manipulation.

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# Conference contributions

## There are several ways to solve an equation

**Jozef Doboš**

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It is better to solve one equation three different ways than to solve three different equations one way. We will show several ways to solve the following equation

$$x^2 + \left(\frac{x}{x+1}\right)^2 = 1.$$

**Keywords.** Solving rational equations.

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# Application of graph theory methods to allocation problems

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Graph theory offers powerful tools for solving problems across various scientific disciplines, ranging from medicine and economics to logistics. One such tool is graph algorithms. In logistics, these algorithms play a key role in solving allocation-related problems. They are applied in areas such as transportation, telecommunications, supply chain management, and resource planning.

This contribution focuses on the application of graph algorithms in solving allocation problems, particularly in the field of logistics. Key topics include the Shortest Path Problem, the Maximum Flow Problem, the Assignment Problem, and the Capacity Planning Problem, the Traveling Salesman Problem, the Partitioning Problem, the Batch Assignment Problem, and the Bin Packing/Container Loading Problem. We discuss fundamental algorithms such as Dijkstra's, Bellman-Ford, Ford-Fulkerson, Edmonds-Karp, and the Hungarian algorithm, among others.

Additionally, we provide numerous examples of real-world applications of these methods in solving practical logistics problems. The insights gained highlight the vast potential of graph algorithms in addressing allocation-type problems.

**Acknowledgement.** This research was funded by VEGA 1/0736/25.

# Tone systems 12-TET and decimal and duodecimal number systems

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In the sixth century BC, the Greek scholar Pythagoras studied vibrations of strings. He dealt with the first three partials of the given tone: the first partial sine function has a frequency of the tone itself, the second one partial (a sine function called the octave) having the tone frequency two times of the first one, the third partial (a sine function called the fifth) has a frequency equal to three times the frequency of the tone. On number three is abbreviated the series of this tone Fourier decomposition.

A sequence of ascending fifths in the positive direction of the real line is called the *Pythagorean circle*.

*Ascending fifth, a positive model F84 of 12-TET*

Under a notion *musical interval* we understand an interval of tones in the multiplicative group of tones. Let us chose an arbitrary fixed tone. We denote by  $\alpha$  a musical interval which starts in this tone and is the conjunction of 12 pure fifth music intervals, where the initial (lower) tone of each subsequent interval is the final higher tone of the following (and the final fifth) interval. Let  $\beta$  be an musical interval created similarly, but as a conjunction of the sequence of 7 pure octaves.

Let starting tones of intervals  $\alpha$  and  $\beta$  be identical. The numerical discrepancy of the resulting tones  $\alpha$  and  $\beta$  expressed in ratio of their first partial frequencies is known as the *Pythagorean comma* interval

$$\pi \stackrel{\text{def}}{=} \frac{\alpha}{\beta} = \frac{(3/2)^{12}}{2^7} = 1.0136\dots > 1.$$

We *psychologically* identify tones which are ends of intervals  $\alpha$  and  $\beta$ . To obtain an equality, all pure fifth musical intervals have to be slightly

diminished (i.e., equally tempered). The tempering can be satisfactorily exact physically done “by hand” with counting beams of vibrating tones. As a result, there are 12 equally tempered tones (also the musical intervals of tones) in one octave.

Using the octave equivalence notion, but in the opposite direction, the resulting tempered one octave is epimorphically extended to all tones of the audio diapason. More precisely, we created the first frequencies of all tones in 12-TET with respect to chosen camertone. Let us denote by F84 the described model of 12-TET based on 84 tones.

*Descending fifths (ascending quarters), a negative model Q60 of 12-TET*

The variant Q60 of 12-TET is derived from 60 music pure quarter intervals (i.e., from 61 tones of descending fifths). Let a conjunction of 12 quarter intervals create an interval  $\gamma$  and a conjunction of 5 octaves create an interval  $\delta$ . Again, having the same first (starting) tone in both intervals, we obtain the same resulting tone.

The difference between the two 12-TET models, F84 and Q60, is that we temper the reverse Pythagorean comma interval: 5 pure octaves are divided into 12 equal quarter intervals. To obtain an equality, we need slightly enlarge (temper) quarters

$$\frac{1}{\pi} = \frac{\delta}{\gamma} = \frac{2^5}{(4/3)^{12}} = 0.9865\dots < 1.$$

After the epimorphic procedure with surjective octave equivalence enlargement analogous to F84 (from one octave to the all audio diapason octaves), we obtain the same result. So, the following theorem holds.

**Theorem 1** *Let  $\mathcal{K}$  be a camertone for the both 12-TET models F84 and Q60. Then sets of tones F84 and Q60 are identical (including their timbres).*

Considering styles of tempering, we have musical intervals based on composites of 1, 2 and 3 (for F84) and based on composites of 1, 2 and 5 (for Q60), respectively. In other words, music in 12-TET would be written in decimal or duodecimal numbers. The second one is most often, in fact all note scores are (isomorphic) written this way. The first style of music coding into natural numbers is rare and non-practical.

The Pythagorean comma notion is commonly known both from the history of mathematics and music.

**Keywords.** Pythagorean comma, camertone, tempering.

**Acknowledgement.** The work was supported by the Slovak Scientific Grant Agency under the project VEGA 2/0134/23.

# To infinity and beyond

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The question of how the sum of infinitely many positive terms can be a finite number was not only a profound philosophical challenge but also an important milestone in understanding the concept of infinity. The contribution is devoted to the presentation of several criteria that answer the question.

**Keywords.** Infinite number series, convergence tests.

**Acknowledgement.** This work was supported by the Slovak Research and Development Agency under contract No. APVV-21-0120 and VEGA 1/0243/23.

# Field theory on the fuzzy onion space

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Combining three fundamental physical constants, the speed of light  $c$ , the reduced Planck constant  $\hbar$ , and the gravitational constant  $G$ , we can form physical quantities with the dimensions of time, energy, length, and so on, that introduce a natural scale for these quantities, called the Planck scale. An interesting quantity is the Planck length,  $l_P \approx 10^{-35}$  m, a very small length scale natural to our universe.

It is reasonable to assume that the spacetime features a kind of a quantum structure, possibly below the Planck scale. This would make the structure impossible to observe directly, as it would require particles with very short wavelengths and in turn very high energies. Our resolution is blurry, fuzzy if the reader will. Despite the lack of experimental possibilities in the

area, it is still possible to study the assumption theoretically and numerically via various models of fuzzy spaces.

The fuzzy onion as described by [1], [2], [3] is a recently proposed model of a three-dimensional space of concentric fuzzy spheres of increasing radii. The model is written in the language of Hermitian matrices, which allows for simple manipulation and numerical simulation. We investigate a scalar field theory on this model numerically using Hamiltonian Monte Carlo methods, comparing the results to both the classical field theory and to a field theory on a single fuzzy sphere.

**Keywords.** Field theory, noncommutative spaces, matrix model.

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# What remains intriguing about the Basel Problem?

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The Basel Problem is a question in mathematical analysis with significance in number theory, concerning the sum of the reciprocals of the squares of all natural numbers. It was first formulated by *Pietro Mengoli* in 1650 and solved by *Leonhard Euler* in 1734. Euler publicly presented the solution on December 5, 1735, at the St. Petersburg Academy of Sciences. Since the problem had resisted the efforts of the leading mathematicians of the time, Euler's solution brought him immediate fame. He later significantly generalized the problem, and his ideas were further developed over a century

later by *Bernhard Riemann* in his famous 1859 paper “On the Number of Primes Less Than a Given Magnitude”, where he defined the zeta function (now named after him) and proved its fundamental properties.

Following Riemann’s notation, we put

$$\zeta(s) := \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad s \in \mathbb{C}.$$

The Basel Problem, therefore, concerns the value of  $\zeta(2)$  and is named after Basel, the hometown of Euler as well as the Bernoulli family, which had attempted unsuccessfully to solve the problem. Although the problem was solved nearly 300 years ago, it remains an intellectual challenge to this day, as evidenced by new verifications appearing in the mathematical literature that employ both classical and advanced techniques. Its significance lies not only in the result itself but also in the journey taken to reach it. The problem illustrates how seemingly intractable mathematical questions can have elegant solutions that pave the way for new insights and applications.

In our contribution, we present the journey of various mathematicians who contributed to solving the Basel Problem. In the bachelor thesis of the second author (under the supervision of the first author) we provided a collection of over 30 different proofs, ranging from approaches using elementary inequalities, single and double integrals, to differentiation under the integral sign. While most of these proofs have been compiled and processed from existing literature, we present several proofs that we have not found in the literature and therefore believe to be new.

**Acknowledgement.** This work was supported by grants APVV-21-0468 and VEGA 1/0657/22.

## Graph-theory based methods for optimization of neural networks

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Optimization plays a pivotal role in enhancing neural network performance, particularly in signal processing, where complex and dynamic data introduce significant challenges.

This contribution focuses exclusively on graph theory-based optimization methods, exploring their potential to address domain-specific constraints such as real-time processing and computational efficiency. We present an in-depth analysis of graph-theoretic approaches for optimizing neural network architectures and training processes. Empirical evaluations, based on FLOPs and MSE metrics, demonstrate that graph-based optimization methods offer scalable, efficient, and environmentally sustainable solutions, significantly reducing computational overhead while maintaining accuracy compared to conventional techniques.

These findings highlight the promising potential of graph-theory-based optimization techniques to revolutionize neural network performance, particularly in signal processing tasks, by balancing computational efficiency, scalability, and environmental sustainability. Future research could expand on these methods to further refine real-time processing capabilities and drive advancements in energy-efficient machine learning systems.

**Acknowledgement.** This research was funded in part by the Slovak Research and Development Agency under contracts No. APVV-22-0508, by the Slovak Grant Agency for Science under grants VEGA 1/0674/23 and VEGA 1/0736/25 and by the Cultural and Educational Grant Agency under grant KEGA 006TUKE-4/2024.

## Structure of trees of primitive Pythagorean triples

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A Pythagorean triple is an ordered triple of positive integers  $(a, b, c)$  such that  $a^2 + b^2 = c^2$ . If  $a, b$  are coprime, then it is called a primitive Pythagorean triple. Clearly, we can view (primitive) Pythagorean triples as (primitive) Pythagorean triangles.

It is known that every primitive Pythagorean triple can be generated from the triple  $(3, 4, 5)$  using multiplication by unique number and order of three specific  $3 \times 3$  matrices, which yields a tree structure. Two such trees were described by Berggren in [1] and Price in [4], respectively. Firstov's results from [2] imply that there are exactly three trees or primitive Pythagorean triples such that  $(a, b, c)$  is a primitive Pythagorean triples

if and only if

$$(a, b, c)^\top = D \cdot (3, 4, 5)^\top,$$

where  $D$  is a product of finite number of matrices from the set  $\{M_1, M_2, M_3\}$ , and  $M_1, M_2, M_3$  are fixed regular  $3 \times 3$  matrices. Moreover, Firstov offered a way of generating the third tree (Firstov's tree).

A new way of viewing the primitive Pythagorean triples is viewing them as coordinates of the points in the 3-dimensional Euclidean space. Considering that each triple has exactly three descendants in the tree, it is natural to ask if these descendants also form a triangle. We prove that the descendants of any primitive Pythagorean triple in Berggren's, Price's and Firstov's tree form a triangle, and we present our results related to these triangles (and these planes). Some of these results can be found in [3].

**Keywords.** Primitive Pythagorean triple, tree of primitive Pythagorean triple.

**Acknowledgement.** Funded by the EU NextGenerationEU through the Recovery and Resilience Plan for Slovakia under the project No. 09I03-03-V05-00008 and grant vvg5-2023-3000.

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# Education of Generation Z – problems, pitfalls, and challenges

## Round table

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Generation Z is growing up in a digital world where technology is an integral part of daily life. This era of rapid technological advancement brings new problems and challenges to education, particularly in mathematics, where traditional methods are no longer always effective. Students of this generation are characterized by their ability to multitask but also face difficulties in maintaining long-term focus and concentration.

The aim of this round table discussion is to explore the problems, pitfalls, and challenges arising from these characteristics of Generation Z and to present innovative methods and tools that can enhance mathematical education. We will discuss specific approaches for using digital technologies, short presentation videos, and interactive visual stimuli to maintain attention and improve the understanding of mathematical concepts. We will also emphasize the importance of creating motivational slogans, gamification, and adapting content to make it not only educational but also engaging and attractive to this dynamic generation.

Our goal is to identify practical tools and approaches that educators can implement to better address the evolving needs and preferences of students, ultimately improving their engagement and success in mathematics.

**Acknowledgement.** We would like to acknowledge VEGA 1/0736/25 and KEGA 006TUKE-4/2024 for funding.

## Elves, wizard and hats

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We will look at several tasks where the main role is played by a wizard and elves. The wizard has the elves in his power and amuses himself by casting spells colored hats to the elves, gives them some rules and clues and elves must guess the color of their own hat... In a way of solutions of such tasks we can find really nice mathematics concerning to epistemic logic, algebra, probability...

**Acknowledgement.** I would like to thank Assoc. Prof. Antonín Slávik from Charles University in Prague for the inspiration.

## Python-based satellite applications in applied mathematics

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The Copernicus satellite system provides vast amounts of Earth observation data, which can be utilized for various scientific and practical applications. Using tools from applied mathematics, the evaluation of satellite images can be optimized through image processing algorithms and data classification methods. The development of a Python satellite application enables

automated data analysis, facilitating fast and efficient decision-making in environmental monitoring, agriculture, and urban planning. The combination of modern programming techniques and mathematical models represents a significant advancement in remote sensing data processing. Our work presents such an application, which documents the construction of the Košice R2 ring road, see Fig. 1.



Figure 1: The Košice R2 ring road.

**Keywords.** Copernicus satellite system, applied mathematics, satellite image evaluation, Python satellite application.

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# What is between a group $\mathbb{Z}_n$ and a ring $\mathbb{Z}_n$ ?

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Algebraic structures on the same set can be compared by using the same set of polynomials that generate those structures. In a group  $(\mathbb{Z}_n, +)$ , we can create less polynomials than in a ring  $(\mathbb{Z}_n, +, \cdot)$ . We are dealing with the question of which sets of polynomials are in this interval.

**Keywords.** Group polynomials, ring polynomials.

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# Support for mathematics teacher education

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In this paper we will present the international project FunThink Erasmus+, which is aimed at developing functional thinking. The project is based on the common idea that mathematics education can be significantly improved by developing functional thinking. Therefore, it is very important to understand the four aspects of functional thinking and the MTSK model [1] on which we build. The aim of the project was to create a digital platform [3] with a learning environment and a number of ideas that can be implemented in mathematics classrooms to promote functional thinking among students and mathematics teachers. An important output is also teacher training courses to enable pre-service and in-service teachers to effectively improve the functional thinking of their students through these learning environments.

Significant part of the outcomes we have created in the FunThink project, together with colleagues from Germany, Poland, the Netherlands and Cyprus, e.g. [5], [6], is offered as part of the training at informal meetings of the Mathematics Teachers Club [4], which is currently under the umbrella of the Digital Transformation of Education School project – DitEdu [2]. The national project aims to create a sustainable ecosystem for supporting the digital transformation of education in the regional education system, which will meet the needs of the digital transformation of society. The relevance and necessity of digital transformation is the main theme of the current Mathematics Teachers Club, where we also address the implementation of digital technologies in education.

**Keywords.** Functional thinking, Mathematics Teachers Club, DitEdu.

**Acknowledgement.** The present work was supported by the Erasmus+ program “FunThink Project” (2020-1-DE01-KA203-005677).

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# Exploring the feasibility of object classification on cave walls using 3D photogrammetric records

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Digital photogrammetry has significantly advanced spatial data analysis, with applications in engineering, environmental monitoring, hydrography, and archaeology. This study explores the feasibility of classifying objects on cave walls using 3D photogrammetric records, focusing on the differentiation between ice and rock formations in a selected section of the Dobšinská Ice Cave. While a complete methodological framework for quantitative classification is not yet established, this research represents a preliminary step toward automated analysis.

Statistical methods are employed to examine the structure and consistency of the collected datasets, utilizing tools such as frequency distributions, boxplots, and correlation coefficients to assess measurement reliability. By comparing data across multiple time periods, we analyze variations in recorded parameters and explore factors influencing classification accuracy. The findings provide insights into the potential of photogrammetric data for object classification and outline key challenges and directions for future research aimed at developing a comprehensive classification methodology.

**Keywords.** 3D photogrammetry, cave environment, object classification, statistical analysis, frequency distribution, correlation analysis, ice formations, rock differentiation.

**Acknowledgement.** This work was supported by the Slovak Grant Agency for Science under grant VEGA 1/0736/25.

# Experimental data fitting using Mittag-Leffler function

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The Mittag-Leffler function, as the generalisation of the exponential function, interpolates between a purely exponential law and a power-law-like behaviour. The properties and computational procedures for evaluating Mittag-Leffler-type functions have been elaborated by many authors, and it became of great use and importance not only for mathematicians, but thanks to its special properties and huge potential for solving applied problems it found its applicability also in the fields such as psychorheology, electrotechnics, economics and econophysics, and for modeling of processes, such as diffusion, combustion, universe expansion, etc. The Mittag-Leffler function arises naturally in the solution of ordinary and partial differential equations of fractional (arbitrary real) order, thus it is widely used in numerical methods for solving such problems. The idea to use the Mittag-Leffler function to model phenomena from different fields is discussed in this contribution, showcasing some applications of experimental data fitting using mathematical models that contain the Mittag-Leffler function in their definition. The results demonstrate the ability of the Mittag-Leffler function to fit data that manifest signs of stretched exponentials, oscillations, or even damped oscillations.

**Keywords.** Mittag-Leffler function, mathematical modelling, data fitting.

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# Mathematical tools of fracture mechanics for students of construction engineering

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Students of engineering mechanics of materials are taught that structures exposed to external loads may substantially change their properties. The very final changes may lead to failure of the structures accompanied by formation of cracks in the materials. Therefore, simulation of those processes is an important part of mechanics of materials and being a naturally complex task provides many challenging problems to be solved by engineers.

The future engineers studying the program of engineering construction are provided many specialised subjects where they learn how to treat with the problem of material changes and degradation using commercial software based on their knowledge of mechanics. Nevertheless, the problems of material damage, and possibly crack nucleation and propagation, are far more complicate that is covered by basic mechanics and computational tools provided in basic mathematical courses at a technical school. Thus, the subject named *Fracture mechanics and plasticity* was included in the study program to cover both mechanical and mathematical aspects of the problems with cracks. As a result, students obtain a theoretical support for computational implementations in those frequently used computer programs.

In the presentation, some of those mathematical aspects, referenced in [2, 3, 4], will be shown for the simplest approach of fracture mechanics, which is denoted as linear elastic fracture mechanics (initiated in [1]). Additionally, the obtained mathematical formulae will be supported by a simple experimental part which makes the formulae to be related to practical problems of engineers.

**Keywords.** Linear elastic fracture mechanics, Airy stress function, fracture modes, analytical methods for partial differential equations.

**Acknowledgement.** This work was supported by the grants: VEGA 1/0307/23 and VEGA 1/0365/25.



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**Program 23. Konferencie košických matematikov****Programme  
of the 23<sup>rd</sup> Conference of Košice Mathematicians****Štvrtok – Thursday 27. 3. 2025****12<sup>00</sup> – Registácia účastníkov – Participants Registration****12<sup>30</sup> – 13<sup>30</sup> Obed – Lunch****14<sup>00</sup> – 14<sup>05</sup> Slávnostné otvorenie konferencie – Conference opening****14<sup>05</sup> – 14<sup>25</sup> Stehlíková (ÚRIVP FBERG TUKE): *Exploring the feasibility of object classification on cave walls using 3D photogrammetric records*****14<sup>30</sup> – 14<sup>50</sup> Kószegyová (ÚMAT PF UPJŠ): *Structure of trees of primitive Pythagorean triples*****14<sup>55</sup> – 15<sup>15</sup> Hovana (KAMaI SjF TUKE): *To infinity and beyond*****15<sup>20</sup> – 15<sup>40</sup> Haluška (MÚ SAV): *Tone systems 12-TET and decimal and duodecimal number systems*****15<sup>40</sup> – 16<sup>10</sup> Občerstvenie – Coffee-break****16<sup>10</sup> – 17<sup>05</sup> Kováčik (KTF FMFI UK Bratislava): *The fuzzy onion*****17<sup>10</sup> – 17<sup>30</sup> Hrmó (FMFI UK Bratislava): *Field theory on the fuzzy onion space*****17<sup>35</sup> – 17<sup>55</sup> Muszka (KLTP LF TUKE): *Python-based satellite applications in applied mathematics*****18<sup>00</sup> – Večera a konferenčný kvíz – Dinner & Conference Quiz**

**Piatok – Friday 28. 3. 2025**

- 7<sup>00</sup> – **Registácia účastníkov – Participants Registration**
- 7<sup>15</sup> – 8<sup>25</sup> **Raňajky – Breakfast**
- 8<sup>30</sup> – 9<sup>25</sup> Brajerčík (KFMT FHPV UNIPO): *Global variational geometry: structures, methods, problems*
- 9<sup>30</sup> – 9<sup>50</sup> Schwartzová (ÚMAT PF UPJŠ): *What is between a group  $\mathbb{Z}_n$  and a ring  $\mathbb{Z}_n$ ?*
- 9<sup>55</sup> – 10<sup>15</sup> Fecková Škrabuláková (ÚRIVP FBERG TUKE): *Application of graph theory methods to allocation problems*
- 10<sup>15</sup> – 10<sup>45</sup> **Občerstvenie – Coffee-break**
- 10<sup>45</sup> – 11<sup>05</sup> Škovránek (ÚRIVP FBERG TUKE): *Experimental data fitting using Mittag-Leffler function*
- 11<sup>10</sup> – 11<sup>30</sup> Jandera (ÚRIVP FBERG TUKE): *Graph-theory based methods for optimization of neural networks*
- 11<sup>35</sup> – 11<sup>55</sup> Slabý (ÚMAT PF UPJŠ): *Support for mathematics teacher education*
- 12<sup>00</sup> – 13<sup>00</sup> **Obed – Lunch**
- 13<sup>00</sup> – 13<sup>55</sup> Hrubý (KAG PřF UP Olomouc): *Scholé*
- 14<sup>00</sup> – 14<sup>40</sup> Lascsáková (KAMaI Sjf TUKE): *Education of Generation Z – problems, pitfalls, and challenges. Round table*
- 14<sup>40</sup> – 15<sup>10</sup> **Občerstvenie – Coffee-break**
- 15<sup>10</sup> – 16<sup>05</sup> Belan (Škola pre mimoriadne nadané deti a Gymnázium Bratislava): *Very old mathematics*
- 16<sup>10</sup> – 16<sup>30</sup> Doboš (ÚMAT PF UPJŠ): *There are several ways to solve an equation*
- 16<sup>35</sup> – 17<sup>30</sup> Snoha (KM FPV UMB): *Dynamical systems: Connecting a math olympiad problem and Benford's law*
- 17<sup>30</sup> – **Večera a spoločenský večer – Dinner & Party**

**Sobota – Saturday 29. 3. 2025**

7<sup>15</sup> – 8<sup>15</sup> **Raňajky – Breakfast**

8<sup>20</sup> – 9<sup>15</sup> Hančová (ÚMAT PF UPJŠ): *Integral transforms in statistics, measurements and data science*

9<sup>20</sup> – 9<sup>40</sup> Vodička (OAMDG SvF TUKE): *Mathematical tools of fracture mechanics for students of construction engineering*

9<sup>40</sup> – 10<sup>10</sup> **Občerstvenie – Coffee-break**

10<sup>10</sup> – 10<sup>30</sup> Hutník (ÚMAT PF UPJŠ): *What remains intriguing about the Basel Problem?*

10<sup>35</sup> – 10<sup>55</sup> Mlynárčik (KM PF KU Ružomberok): *Elves, wizard and hats*

11<sup>00</sup> – 11<sup>05</sup> **Záver konferencie – Conference closing**

11<sup>10</sup> – **Obed – Lunch**

## Zoznam účastníkov – Participants list

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