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Pobočka Košice

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Ústav matematiky

Fakulta baníctva, ekológie, riadenia a geotechnológií TUKE
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22. Konferencia košických matematikov

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Predhovor

Vážení účastníci Konferencie košických matematikov a ostatní čitatelia!

V rukách práve držíte zborník abstraktov príspevkov z Konferencie košických matematikov a spolu s ním sa zoznamujete s 22. kapitolou histórie tohto podujatia. Myšlienka zorganizovať konferenciu tohto typu vznikla na pôde košickej pobočky Jednoty slovenských matematikov a fyzikov pri Slovenskej akadémii vied koncom deväťdesiatych rokov minulého storočia. Cieľom bolo poskytnúť platformu pre stretnutia ľudí, ktorí sa profesionálne venujú matematike, žijúcim na východe Slovenska – pre učiteľov, vedcov, aplikovaných matematikov... Šlo o iniciatívu založiť pravidelné fórum, na ktorom môžu diskutovať s ostatnými kolegami, zdieľať svoje radosti a starosti súvisiace s prácou; hľadať riešenia pedagogických, didaktických a vedeckých problémov.

Aj keď nevšedné udalosti posledných rokov zanechali nejaké diery na línii pravidelnosti usporiadavania Konferencií košických matematikov, možno povedať, že sú už tradičným podujatím organizovaným v Herľanoch v jarných dňoch. Už od začiatku mali organizátori predstavu o založení serióznej konferencie, kde by bol kvalitný obsah garantovaný najmä pozvanými prednáškami. Preto boli na toto fórum pozývané osobnosti, ktoré sú dobre známe vo svojom vedeckom či pedagogickom prostredí, ktorých múdrosť a skúsenosti majú potenciál obohatiť široké publikum. Ich schopnosti môžu povzbudiť najmä mladých kolegov a doktorandov na začiatku ich profesionálnej kariéry. Práve preto by mala byť táto konferencia adresovaná aj im a poskytnúť im niečo ako prvú bránu k vedeckým prezentáciám, besedám, diskusiám. Sme veľmi radi, že počas viac ako 20-ročnej histórie konferencie mnohí dnes už veľmi úspešní kolegovia mali svoje prvé verejné alebo vedecké vystúpenie práve na Konferencii košických matematikov.

Pozvanie predniesť prednášku na 22. Konferencii košických matematikov prijali: prof. RNDr. Katarína Cechlárová, DrSc. (Prírodovedecká fakulta, Univerzita P. J. Šafárika v Košiciach, Košice, Slovenská republika), Mgr. Zdenka Jeremiašová (ZŠ Lesnícka 1, Prešov, Slovenská republika), doc. Mgr. Richard Kollár, PhD. (Fakulta matematiky, fyziky a informatiky, Univerzita Komenského v Bratislave, Bratislava, Slovenská republika), Mgr. Tereza Kovářová, Ph.D. (Fakulta elektrotechniky a informatiky, Vysoká škola báňská – Technická univerzita Ostrava, Ostrava – Poruba, Česká republika), RNDr. Renáta Ujháziová, PhD. (IBM Slovensko, spol. s r.o., Košice, Slovenská republika), prof. Ing. Roman Vodička, PhD. (Stavebná fakulta, Technická univerzita v Košiciach, Košice, Slovenská republika) a RNDr. Petra Vondráková, Ph.D. (Fakulta elektrotechniky a informatiky,

Vysoká škola báňská – Technická univerzita Ostrava, Ostrava – Poruba, Česká republika).

Milí účastníci konferencie, teší nás, že vás môžeme privítať na Konferencii košických matematikov. Veríme, že sa tu budete cítiť príjemne a načerpáte tu množstvo motivácie do ďalšej tvorivej práce.

Editori: Ján Buša
Erika Fecková Škrabuľáková
Andrea Feňovčíková

Editorial

Dear participants of the Conference of Košice Mathematicians and other readers!

In your hands you are currently holding a booklet of abstracts of contributions presented at a Conference of Košice Mathematicians. Hereby you are getting acquainted with the 22nd chapter of the history of this event. The idea to organize a conference of this type originated at the Košice's branch of the Union of Slovak Mathematicians and Physicists by Slovak Academy of Science at the end of the nineties of the last century. The goal was to provide a platform for meetings of people living in the eastern part of Slovakia who are professionally engaged in mathematics – for teachers, scientists, applied mathematicians... There was an initiative to establish a regular forum where they can discuss with other colleagues, share their work-related joys and worries; to look for solutions of pedagogical, didactic and scientific problems.

Even though the unusual events of recent years have left some holes in the timetable regularity of Conferences of Košice Mathematicians, it can be said that they are already a traditional event organized in Herľany in spring. From the very beginning, the organizers had the idea of a serious conference establishment, where quality content would be guaranteed, especially through invited lectures. Therefore, personalities who are well known in their scientific or pedagogical environment, whose wisdom and experience have the potential to enrich a wide audience, were invited to this forum. Their skills can especially encourage young colleagues and PhD students at the beginning of their professional careers. That is why this conference should also be addressed to them and provide them with something like the first gateway to scientific presentations, talks, and discussions. We are very happy that during the more than 20-years history of the conference, many of our now very successful colleagues had their first public scientific presentation at the Conference of Košice Mathematicians.

The invitation to give an invited lecture at the 22nd Conference of Košice Mathematicians accepted: prof. RNDr. Katarína Cechlárová, DrSc. (Faculty of Science, Pavol Jozef Šafárik University, Košice, Slovak Republic), Mgr. Zdenka Jeremiašová (Primary School Lesnícka 1, Prešov, Slovak Republic), doc. Mgr. Richard Kollár, PhD. (Faculty of Mathematics, Physics and Informatics, Comenius University, Bratislava, Slovak Republic), Mgr. Tereza Kovářová, Ph.D. (Faculty of Electrical Engineering and Computer Science, VSB – Technical University Ostrava, Ostrava – Poruba, Czech Republic), RNDr. Renáta Ujháziová, PhD. (IBM Slovakia, Ltd., Košice,

Slovak Republic), prof. Ing. Roman Vodička, PhD. (Faculty of Civil Engineering, Technical University of Košice, Košice, Slovak Republic) and RNDr. Petra Vondráková, Ph.D. (Faculty of Electrical Engineering and Computer Science, VSB – Technical University Ostrava, Ostrava – Poruba, Czech Republic).

Dear participants of the conference, we are pleased to be able to welcome you at the Conference of Košice Mathematicians. We believe that you will feel comfortable here and that you will draw here a lot of motivation for further creative work.

Editors: Ján Buša
Erika Fecková Škrabuľáková
Andrea Feňovčíková

Obsah – Contents

Invited lectures

On three applications of linear programming

Katarína Cechlárová

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In this contribution we present the power of linear (in)equalities in solving nontraditional problems. Using the following examples we demonstrate how a mathematical model of a real situations was created, its computational complexity analysed, an algorithmic solution proposed, and numerical simulations with real and generated data performed:

- assignment of trainee teachers to schools,
- construction of fair schedules of Young Physicists Tournament,
- election candidate nomination.

Keywords. Integer programming, computational complexity, numerical simulation, applications.

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Specifics of education of pupils from the Roma community in primary school

Zdenka Jeremiašová

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In this contribution the observations from the long-term practice of education of pupils from the Roma community in primary school are presented. Here we analyze these observations from several perspectives, namely, from the point of view of mathematics teaching, from the point of view of the classroom teacher, and from the point of view of interpersonal and inter-community relations between the majority and this minority.

While speaking about the mathematics teaching point of view, it is mainly about the usage of appropriated methods of education and work itself, didactic aids, staying with students in class, their homeworks and own preparation for the study process itself.

From the point of view of a class teacher, we analyze specific problems related to the language barrier between teacher and parents, their ability to understand expressed topics and the lack of understanding of their duties as the legal representatives of the pupil.

The conclusion of the contribution is devoted to more or less disturbed interpersonal relations and prejudices from one hand side or the other. It also deals with tips on how to break down these prejudices, how to work not only with students and their parents, but also with field workers working with the community in their place of residence.

Self-organization as a multiscale absorbing Markov process

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We explore self-organization in a stochastic dynamical system using rigorous and numerical multiscale analysis. The system, at the microscale, describes lane-switching of individuals moving in opposite directions on a circular multi-lane track. At the mesoscale, some system variants are modeled as absorbing Markov chains, while others are not. However, at the macroscale, all variants reduce to an absorbing Markov chain model, ensuring self-organization within a finite time. Our findings show a constant upper limit for the mean time to self-organization regardless of the number of individuals in the system in variants with mesoscopic Markovian properties. That aligns with a simplified Poisson process model following the Mass action law. Conversely, non-Markovian variants on the mesoscale exhibit longer self-organization times, although numerical simulations indicate that some of them also achieve bounded self-organization times. We also identify a non-Markovian variant that demonstrates crowd formation tendencies, leading to prolonged time of self-organization. For another variant, we confirm linear growth in self-organization time observed in the literature for simple lane-switching models. Our study provides a framework for applying absorbing Markov chain theory in self-organization, particularly in systems involving conflict resolution mechanisms.

Keywords. Self-organization, absorbing Markov chain, crowd formation, traffic model, mass action law.

Acknowledgement. This work has been supported by the Scientific Grant Agency of the Slovak Republic under grants no. 1/0521/20 and 1/0755/19, and by the Slovak Research and Development Agency under contract no. APVV-18-0308.

Generative AI and large language models (LLMs)

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Generative AI and large language models have revolutionized natural language processing, enabling machines to generate coherent and contextually relevant text. In this presentation, we delve into the intricacies of these sophisticated models and explore their capabilities while showcasing their real-world impact on artificial intelligence.

We will discuss how generative AI systems, particularly large language models like GPT (Generative Pre-trained Transformer), operate by leveraging deep learning architectures and vast amounts of data to generate human-like text. Furthermore, we will demonstrate the critical role of mathematics in evaluating these models, examining metrics such as BLEU scores, ROUGE scores, etc., to quantify their performance and assess their quality.

This presentation aims to provide a comprehensive understanding of generative AI and large language models, elucidating their potential and guiding effective utilization in various domains. This is your chance to embark on a captivating journey into the future of AI!

Computational fracture mechanics: Models with internal variables

Roman Vodička

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Materials of engineering structures, when exposed to external loads, may significantly change its properties. Ultimately, the changes could lead to failure of the structures. The failure is frequently accompanied by nucleation and propagation of cracks in the materials: either directly inside the material or along the material component interfaces. Therefore, a computational simulation of those cracks is an important part of mechanics of materials and, due to its complexity, it is a challenging task.

Computational methods for fracture modelling follow two basic directions of how they treat the cracks. First, the crack is (naturally) considered as a discontinuity of the structure geometry, or, second, the possible discontinuity is diffused over structure components by defining changes in material and producing so called regularised cracks. The material changes are determined by internal variables and usually they are referred to as damage.

In the presentation, a computational approach for analysis of regularised cracks developed by the author is described, implemented and tested in a scheme which utilises a phase-field fracture model [2] for solids simultaneously with an independent application of an adhesive contact model for interface damage [1]. A variationally based approach for a computer simulation [3] is put into effect in an in-house MATLAB code which uses the finite element method and algorithms of sequential quadratic programming. Although, the tested solids represent academic examples of multi–domains structural elements containing inhomogeneities, they simulate actual experimental observations.

Keywords. Phase-field fracture, interface damage, finite element method, sequential quadratic programming, staggered approach.

Acknowledgement. This work was supported by the following grants: VEGA 1/0363/21 and VEGA 1/0307/23.

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How to revive teaching, create tailored tests, or simply practice mathematics with Math4U

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Math4U (`math4u.vsb.cz`) is a multilingual portal offering students applications to practice all areas of secondary school mathematics. It provides teachers with effective tools for creating tests in several output formats and additional options to revive math lessons. Users of the Math4U portal have access to an extensive database of questions covering the entire secondary school math curriculum. All content is available in five languages: English, Spanish, Slovak, Polish, and Czech. In our talk, we will introduce you to the Math4U portal and demonstrate its functionalities, which can be used advantageously by both teachers and students. The portal consists of five main sections (apps): Student, Student Easy, Class, Teacher, and Test4U.

The Student and Student Easy apps are primarily designed for students to practice mathematics. The Student app works with a database of 6,000 multiple-choice questions, while the Student Easy app utilizes a database of 2,000 easy questions, solvable without the need for pen and paper.

The Teacher app (Math4Teacher) allows teachers to quickly create tests in different languages and in three output formats: printable PDF, interactive PDF suitable for use on an interactive whiteboard or in a computer lab, and HTML. Teachers can select multiple-choice questions from a database of 6,000 questions and generate tests in these output formats from the selected questions. Additionally, we mention the Test4U app, which complements the Math4Teacher app and allows students to access teacher-created HTML tests. This application also provides immediate feedback to teacher on students' test performance, offering evaluations of individual student answers and statistics on answers to individual questions instantly. Teachers can utilize this information to identify and effectively address problematic parts of the curriculum.

The Class section of the portal contains 150 ready-made quizzes and practice games. These interactive PDF quizzes are automatically evaluated upon completion, and the aim is to find the correct question-answer pairs.

The portal is not only designed for students and teachers of secondary schools but also for math practice in the first years of study at technical universities. It can be used successfully to practice material covered in pre-calculus courses involving trigonometry, algebraic, exponential, or logarithmic functions. Math4U also includes more than 500 questions on Single Variable Calculus.

Acknowledgement: The authors are supported by the Erasmus+ programme of the European Union under grant agreement No. 2022-1-CZ01-KA220-SCH-000086821.

Conference contributions

Dynamic maximum entropy approximation for understanding complex stochastic dynamics

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Realistic models of biological processes typically involve interacting components on multiple scales, driven by changing environment and inherent stochasticity. Such models are often analytically and numerically intractable. We will present a dynamic maximum entropy (DME) method, which approximates the full dynamics by a low-dimensional system of ODEs describing the system. The method circumvents the problem of closing an infinite hierarchy of moment equations by choosing a suitable set of functions, whose averages are followed in time. Using a quasi-stationarity assumption, the problem reduces to low-dimensional dynamics, which shows great accuracy even in non-stationary problems. While estimating accuracy of the method is still an open problem (we will indicate why), we will present a couple of applications ranging in complexity where:

1. the approximation recovers the exact solution,
2. the approximation is accurate but not exact,
3. the approximation fails.

From curvature to tortuosity

Miloslav Cisko

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Tortuosity is an under-researched concept, but it could have a relatively wide practical application. It is a relatively new concept, as all works dealing with this concept come from the end of the 20th and the beginning of the 21st century [2, 3, 4, 5]. However, most of these works are devoted to practical use, whether it is the tortuosity of curves representing rivers, or the tortuosity of animal movement in the wild, or the tortuosity of human blood vessels.

Therefore, this work was created as an effort to create a definition that would better capture tortuosity. So that it can capture all the curves, so that it is invariant to all important properties and so that the resulting values correspond to the ideas. With possible ideas of the generalisation of it.

In the first part, we will deal with the basic theory of curves, see [1, 6]. We will go through the derivative of the curve, the tangent to the curve, the formula for calculating the length of the curve, reparameterization, and natural reparameterization. Finally, we come to higher derivatives and the key concepts of the first part, the curvature of a curve.

In the second part, we will look at the currently known approaches to the concept of tortuosity. Then we will take a closer look at the curve integral, necessary for our definition. Subsequently, we state our definition and prove some basic properties such as invariance to shift, reparameterization, and rotation of the curve.

Keywords. Curves, curvature, line integral, tortuosity.

Acknowledgement. The present work was supported by the Slovak Research and Development Agency under Contract no. APVV-21-0468.

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What we learn about quadratic equations

Jozef Doboš

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Some interesting topics will be presented, what we could learn about quadratic equations.

Acknowledgement. This work was supported by KEGA grant No. 004KU-4/2022 Slovak Mathematics II – Life Patterns for Future Generations.

Utilization of the graph theory tools in order to increase the information content of data

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Graph theory tools are widely used in technical disciplines and other scientific fields. This is evidenced, for example, by a large number of publications in which knowledge from graph theory is applied to the study of various phenomena from technical and technological practice. The applications of graph theory in research conducted at the Technical University of Košice have brought fruit, for example, in connection with the acquisition and processing of land resources, multicriterial classification of spatial data, in connection with research in the field of finance, banking and economics, in the field of neural networks and data mining, in solving several problems from the field of logistics, and even those connected with human health and the like. In this contribution, we provide a review of applications of graph theory used in the works of researchers from Technical University of Košice with the aim of strengthening the motivation for basic and applied investigation in this area.

Acknowledgement. This research was funded by VEGA 1/0264/21 and KEGA 006TUKE-4/2024.

Technologies in mathematical education. Round table – part 2

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A method of roundtable discussion is based on an organized conversation between moderator and speakers that bring a variety of perspectives to a subject. The audience may simply observe or participate. The round table method has been known at least since 12^{th} century. The method is simple but highly effective. Everyone understands the value of two-way communication. The structured environment helps to increase its efficiency.

In the second part of the round table, we will deal with specific problems of teaching process of mathematics connected with distance learning, teaching and communication, information technologies and alike. We will discuss the utilization of suitable software and tools in education process and experiences with teaching mathematics via software support. We will search for motivation to study mathematics via suitable examples of its utilization in connection with automatization, AI,... Last but not least we will discuss the way of writing mathematical texts.

We believe that stimulating such discussion will bring not only the overview on used strategies of teaching mathematics and the software used, but also some new inspirations for participants.

Acknowledgement. We would like to acknowledge VEGA 1/0264/21, VEGA 1/0532/22, KEGA 006TUKE-4/2024 and KEGA 012TUKE-4/2022 for funding.

Modular lattice – a short memory of the centenary of the birth of Ján Jakubík

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Professor Ján Jakubík (8. 10. 2023, Dudince – 24. 11. 2015, Košice) is one of the important world mathematicians. The main research field throughout his whole career, has been the theory of ordered algebraic structures, especially lattice ordered groups and related algebras. He published more than 300 scientific papers. In addition, he belongs to people who were at the birth of the Technical University and the P. J. Šafárik University in Košice and trained multitudes of experts at these schools.

Besides other distinctions, professor Ján Jakubík was awarded by the 1st Class Ľudovít Štúr Order by the State President of the Slovak Republic in 2008, see [1].

The notion of lattice and its basic properties are a part of the foundations of higher mathematical education today. We are convinced that the result of [2] which follows should be widely known in homeland of Ján Jakubík. For alternative proof see [3].

Let L be a lattice. A sublattice M of L is called *cover-preserving*, if x covers y in M implies x covers y in L for all $x, y \in M$.

A lattice L is said to be

- *cell* if there are $u, v \in L$, $u \neq v$ and chains $C, C' \subset L$ such that
  (a) $L \neq \{u, v\}$, $C \cup C' = L$ and $C \cap C' = \{u, v\}$,
  (b) if $x \in C \setminus \{u, v\}$ and $y \in C' \setminus \{u, v\}$, then $x \vee y = u$ and $x \wedge y = v$.
- 4-*cell* if every cover-preserving sublattice M of L satisfies the condition that if M is a cell, then M is isomorphic to the four-element Boolean lattice.
- *discrete* if every bounded chain is finite.

Note that every finite lattice is discrete.

Theorem 1 (Jakubík, 1975) *Let L be a discrete lattice. Then the following conditions are equivalent:*

1. *L is modular,*
2. *L is 4-cell lattice and lattices S_7, S_7^*, see Fig. 1, are not cover-preserving sublattices of L.*

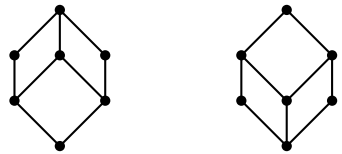

Figure 1: Lattices S_7 and S_7^*.

Keywords. Ján Jakubík, modular lattice, cell, cover-preserving.

Acknowledgement. This work was supported by the Slovak Scientific Grant Agency VEGA 2/0104/24.

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On a function monotonicity problem

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Monotonicity of a function is an important property that describes behaviour of the function depending on its argument. Investigating the monotonicity of the function is often a part of mathematical analysis and is crucial in many applications of mathematics, physics, economics, and other fields. We will focus on functions which can be expressed as a ratio of two functions f and g defined on some interval $[a, b]$ such that $r := f/g$ has sense.

Keywords. Monotonicity, ratio, derivative.

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Modern trends in linear prediction

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The following contribution introduces two novel types of linear prediction (LP) models that outperform the standard short-term and long-term linear prediction models. The proposed approach involves a vector of seasonal weights estimated based on basic statistical values. These weights act on the previous samples used for the prediction, resulting in better performance of the developed LP models in terms of the chosen metrics, such as R-squared, root mean squared error (RMSE), mean squared error (MSE), and prediction gain (PG), indicating their superior accuracy and predictive power. Furthermore, based on experiments, an interesting observation from

an application perspective is that the proposed short-term linear prediction model enhanced using seasonal weights achieved the same fitting performance in terms of R-squared as the standard long-term linear prediction model but with better performance in terms of RMSE, MSE, and PG, even when using fewer prediction parameters. Thus, having fewer prediction parameters results in speeding up the prediction and further transmission of the coded data, which can be of great importance in the field of digital signal processing.

Keywords. Linear prediction, seasonal weights, signal processing.

Acknowledgement. This research was funded in part by the Slovak Research and Development Agency under contract No. APVV-18-0526 and contract No. APVV-22-0508, by the Slovak Grant Agency for Science under grant VEGA 1/0674/23, and by the Cultural and Educational Grant Agency under grant KEGA 006TUKE-4/2024.

On 2-connected graphs with small circumference

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The circumference $\mathrm{cir}(G)$ of a 2-connected graph G is the length of its longest cycle. The problem of characterization of all 2-connected graphs is, in general, very difficult. To see this, note that Hamiltonian graphs on $n > 1$ vertices have circumference n.

We show that for some small k, a characterization of all 2-connected graphs G with $\mathrm{cir}(G) = k$ is possible. We also formulate some open questions concerning these families of graphs.

Education of communities. Round table – part 1

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The round table method has been known for centuries. It is a very simple but highly effective method. Round tables use two-way communication in a structured environment. Ideas are not taught, but facilitated. The facilitator encourages people to share and allows them to learn and grow by their personal example.

In the first part of the round table, we will deal with pressing problems of the teaching process in primary, secondary and higher schools. We will discuss the specifics of the education of different communities. We will focus on the problems of teaching mathematics in a foreign language and teaching mathematics in Slovak language for foreigners. We will not forget questions regarding the sufficiency or lack of study materials, books, supporting literature and other study aids.

We believe that stimulating the discussion on these topics will not only bring interesting views of the participants, but also outline possible solution strategies.

Acknowledgement. We would like to acknowledge VEGA 1/0264/21, VEGA 1/0532/22, KEGA 006TUKE-4/2024 and KEGA 012TUKE-4/2022 for funding.

On anti-A-walks in plane graphs

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Given a graph G and its rotation system $\mathcal{R} = \bigcup_{v \in V(G)} R(v)$, where $R(v)$ is a cyclic clockwise ordering of edges incident with v, a walk $(v_0, e_1, v_1, \ldots, e_k, v_k)$ in G is an *anti-A-walk* if, for every $i \in \{1, \ldots, k-1\}$, the edges e_i, e_{i+1} are not successive in $R(v_i)$. A particular example of such a walk is earlier known *cut-through trail* in a 4-regular plane graph (where the edges e_i, e_{i+1} are opposite in $R(v_i)$), see [1], [3].

Focusing on study of properties of anti-A-walks in plane graphs, we address the question of their existence between each pair of vertices. While this is not always possible in 4-regular plane graphs (resp. plane graphs of minimum degree at least 4) in the sense of existence of cut-through paths (see [4]), the problem is open (even with anti-A-walks) for plane graphs of minimum degree 5 (resp. 5-regular ones, [2]). We developed a set of tools (in Maple computer algebra system [5]) for testing path anti-A-connectivity of 5-regular plane graphs; in addition, for finding the shortest anti-A-paths (resp. trails) between two vertices in a graph with given rotation system, we used integer linear programming approach. We also present large families of polyhedral graphs of minimum degree 5 (with exponentially many members for fixed number of vertices) in which every pair of vertices is connected by an anti-A-path.

Keywords. Anti-A-walk, cut-through path, plane graph.

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On irregular labelings of graphs

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Various types of graph labelings have been intensely studied over the last few decades. One of the most challenging tasks in the theory of labeled graphs is to prove that it is possible to label the edges of any graph other than K_2 with consecutive integers from 1 to k so that the sum of integers at each vertex is different. Such a labeling is called irregular. The minimal k for which such labeling exists is known as the strength of the graph.

In this talk, we will focus on two types of irregular labelings. We will discuss the face irregularity strength of type (α, β, γ) for plane graphs. We will present some lower and upper bounds for this graph invariant and determine the precise values of this parameter for certain families of plane graphs.

Additionally, we will explore the reflexive edge strength for the Cartesian product of two cycles and for the Cartesian product of two paths.

Keywords. Irregular labeling, face irregularity strength of type (α, β, γ), reflexive edge strength.

Microlearning in engineering education

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Microlearning is one of the new approaches to modern education. This contribution presents the main pillars of microlearning, its historical perspective, and its benefits for the educational process in the 21st century.

Implementing microlearning in the mathematics course requires redesigning the course and creating specific electronic materials for the learning and teaching process. This contribution brings our experiences with its implementation to the Mathematics 1 course at the Faculty of Mining, Ecology, Process Control and Geotechnologies of the Technical University of Košice.

Keywords. Microlearning, education process, engineering education.

Acknowledgement. The present work was supported by the Scientific Grant Agency (VEGA) under grant VEGA 1/0674/23, the Cultural and Educational Grant Agency MŠVVaŠ SR (KEGA) under grant 006TUKE-4/2024, and by the Slovak Research and Development Agency (APVV) under grant APVV-22-0508.

Proposal of principles of a classification method for classifying rocks based on attributes of the vibration signal generated during rock drilling

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The research aimed to develop a method for classifying rocks based on vibration signals. The paper presents procedures and results of individual steps of the design. The proposed method currently works for four types of rocks: granite, concrete, limestone, and andesite. Descriptive statistics indicators, ANOVA, Cluster dendrogram based on Euclidean distance, and decision tree tools from machine learning were used to create the method. The paper presents the results of attribute selection and length of vibration signal for rock classification purposes and verification of their suitability for rock classification, the proposal of attribute combinations to ensure that the Euclidean distance between objects of different rocks is statistically more significant than the distance between objects of the same rock, a classification rule, which was created for the train set and verified on test set data. The method's efficiency was 100 % on the test data.

Keywords. Classifying rocks, vibration signals, decision tree.

Acknowledgement. This work was supported by the Slovak Grant Agency for Science under grant VEGA 1/0264/21 Application of modern methods in the analysis and modeling of technological and other processes used in the acquisition and processing of earth resources in order to optimize them.

On a specific vertex coloring of graphs

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Graph coloring is an assignment of labels to elements of a graph subject to certain constraints. These labels are traditionally called colors. Coloring the vertices of a graph such that no two adjacent vertices are of the same color is called proper vertex coloring. Proper vertex coloring is called proper open interval vertex coloring if colors used on the neighbors of an arbitrary vertex form an integer interval. Proper vertex coloring is called proper closed interval vertex coloring if the set of colors used on the closed neighborhood of every vertex forms an integer interval.

In this contribution we present the recent results on both proper closed interval vertex coloring and proper open interval vertex coloring. Regarding proper closed interval vertex coloring, we show that there are k-chromatic graphs, $k \geq 4$, that do not admit a proper closed interval vertex coloring. Moreover, we provide multiple constructions of proper closed interval vertex non-colorable graphs.

Acknowledgement. This research was funded by the Slovak Grant Agency for Science under grant VEGA 1/0674/23.

Important personalities of Slovak mathematics

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There are several mathematical centres in Slovakia that are important impulses for the development of mathematics on an international scale. These centres concentrate many world-class mathematicians who can be an inspiration for the younger generation. Within the pilot project Personalities of Slovak mathematics – life models for future generations, we have processed the Mikuláš mathematical school and 6 prominent personalities (doc. Frič, prof. Hejný, prof. Bukovský, prof. Dvurečenskij, prof. Jendroľ, prof. Šedivý). At present, we are continuing the above project and working on the second important centre of Slovak mathematics – the Košice Mathematical School. In the first part we have already presented the history of the Košice Mathematical School of Set Theory and Topology – Seminar of prof. Lev Bukovský, [1]. Now we are preparing the second part of it – the Košice School of Discrete Mathematics, which is being created in cooperation with prof. Jendroľ and prof. Madaras. The last part will be a presentation of the history of the Conference of Košice Mathematicians in Herľany, which we are working on in collaboration with Erika Fecková Škrabuľáková and Ján Buša.

The aim of the talk is to present the results of the second project so far, which captures important Slovak mathematicians (prof. Čižmár, prof. Cechlárová, doc. Velichová, [2, 3]) as well as maps of the second important mathematical workplace in Slovakia – the Košice Mathematical School.

Keywords. History of Košice mathematics school, personalities of Slovak mathematics, education.

Acknowledgement. This work was supported by KEGA grant No. 004KU-4/2022 Slovak Mathematics II – Life Patterns for Future Generations.

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Program 22. Konferencie košických matematikov

Programme of the 22nd Conference of Košice Mathematicians

Štvrtok – Thursday 25. 4. 2024

12^{00} – **Registácia účastníkov – Participants Registration**

12^{30} – 13^{30} **Obed – Lunch**

14^{00} – 14^{05} **Slávnostné otvorenie konferencie – Conference opening**

14^{05} – 14^{25} Madaras (ÚMAT PF UPJŠ): *On anti-A-walks in plane graphs*

14^{30} – 14^{50} Jendroľ (ÚMAT PF UPJŠ): *On* 2*-connected graphs with small circumference*

14^{55} – 15^{15} Stehlíková (ÚRIVP FBERG TUKE): *Proposal of principles of a classification method for classifying rocks based on attributes of the vibration signal generated during rock drilling*

15^{20} – 15^{40} Cisko (ÚMAT PF UPJŠ): *From curvature to tortuosity*

15^{40} – 16^{10} **Občerstvenie – Coffee-break**

16^{10} – 17^{05} Jeremiašová (ZŠ Lesnícka Prešov): *Specifics of education of pupils from the Roma community in primary school*

17^{10} – 18^{00} Lascsáková (KAMaI SjF TUKE): *Education of communities. Round table – part 1*

18^{00} – **Večera a konferenčný kvíz – Dinner & Conference Quiz**

Piatok – Friday 26. 4. 2024

7^{00} – **Registácia účastníkov – Participants Registration**

7^{15} – 8^{15} **Raňajky – Breakfast**

8^{20} – 9^{15} Kollár (KAMŠ FMFI UK Bratislava): *Self-organization as a multiscale absorbing Markov process*

9^{20} – 10^{15} Cechlárová (ÚMAT PF UPJŠ): *On three applications of linear programming*

10^{15} – 10^{45} **Občerstvenie – Coffee-break**

10^{45} – 11^{40} Ujháziová (IBM Slovensko): *Generative AI and large language models (LLMs)*

11^{45} – 12^{05} Jandera (ÚRIVP FBERG TUKE): *Modern trends in linear prediction*

12^{10} – 12^{30} Boďová (KMANM FMFI UK Bratislava): *Dynamic maximum entropy approximation for understanding complex stochastic dynamics*

12^{30} – 13^{30} **Obed – Lunch**

13^{30} – 13^{50} Hovana (KAMaI SjF TUKE): *On a function monotonicity problem*

13^{55} – 14^{15} Šárošiová (ÚRIVP FBERG TUKE): *On a specific vertex coloring of graphs*

14^{20} – 14^{40} Ovais (UET Lahore Pakistan): *On irregular labelings of graphs*

14^{40} – 15^{10} **Občerstvenie – Coffee-break**

15^{10} – 16^{35} Vondráková, Kovářová (KAM FEI VŠB – TU Ostrava): *How to revive teaching, create tailored tests, or simply practice mathematics with Math4U*

16^{40} – 17^{00} Pócsová (ÚRIVP FBERG TUKE): *Microlearning in engineering education*

17^{10} – 18^{00} Fecková Škrabuľáková (ÚRIVP FBERG TUKE): *Technologies in mathematical education. Round table – part 2*

18^{00} – **Večera a spoločenský večer – Dinner & Party**

Sobota – Saturday 27. 4. 2024

7^{15} – 8^{15} **Raňajky – Breakfast**

8^{20} – 9^{15} Vodička (OAMDG SvF TUKE): *Computational fracture mechanics: Models with internal variables*

9^{20} – 9^{40} Doboš (ÚMAT PF UPJŠ): *What we learn about quadratic equations*

9^{40} – 10^{10} **Občerstvenie – Coffee-break**

10^{10} – 10^{30} Fecková Škrabuľáková (ÚRIVP FBERG TUKE): *Utilization of the graph theory tools in order to increase the information content of data*

10^{35} – 10^{55} Halušková (MÚ SAV): *Modular lattice – a short memory of the centenary of the birth of Ján Jakubík*

11^{00} – 11^{20} Tkačik (KI PF KU Ružomberok): *Important personalities of Slovak mathematics*

11^{20} – 11^{25} **Záver konferencie – Conference closing**

11^{25} – **Obed – Lunch**

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