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Predhovor

Vážení účastníci Konferencie košických matematikov a ostatní čitatelia!

Udalosti posledných rokov poznačených pandémiou nás prinútili žiť dlho v izolácii. Osobné stretnutia boli nemožné, organizovanie masových akcií nemysliteľné. . . Fungujúc v okruhu obmedzení bolo ťažké plánovať, predvídať budúce udalosti, odhadovať možné scenáre ich vývoja a pružne na ne reagovať. Neustále sa meniace opatrenia nás nútili opakovane odkladať a presúvať Konferenciu košických matematikov, čo viedlo k tomu, že na jubilejnú dvadsiatu konferenciu organizovanú v roku 2019 nadväzuje ďalšia až v roku 2023.

Krušné mesiace a roky nám ukázali dobré zrkadlo našej spoločnosti a zdôraznili dôležitosť vedeckého výskumu, medziludských osobných kontaktov i prepojenia výskumu a praxe. O to viac dnes v spoločnosti rezonuje nutnosť socializácie a udržiavania spoločenských kontaktov, nakoľko sociálne prepojenie nám nielen prináša potešenie, ale ovplyvňuje aj naše dlhodobé zdravie. Vďaka online platformám sme síce v uplynulom období mali možnosť zostať v kontakte, no tieto nedokázali nahradiť stretnutia tvárou v tvár. Z vlastných skúseností vieme, že i vedecký výskum a bádanie býva v prípade priameho kontaktu plodnejšie. Z týchto dôvodov si obzvlášť vážime, milí účastníci konferencie, že vás môžeme na Konferencii košických matematikov privítať osobne.

Myšlienka zorganizovať konferenciu tohto typu vznikla na pôde košickej pobočky Jednoty slovenských matematikov a fyzikov koncom deväťdesiatych rokov minulého storočia. Cieľom bolo poskytnúť platformu pre stretnutia ľudí, ktorí sa profesionálne venujú matematike žijúcich na východe Slovenska – pre učiteľov, vedcov, aplikovaných matematikov. . . Šlo o iniciatívu založiť pravidelné fórum, kde môžu diskutovať s ostatnými kolegami, zdieľať svoje radosti a starosti súvisiace s prácou; hľadať riešenia pedagogických, didaktických a vedeckých problémov. Už od začiatku tu bola predstava serióznej konferencie, kde by bol kvalitný obsah garantovaný najmä pozvanými prednáškami. Preto boli na toto fórum pozývané osobnosti, ktoré sú dobre známe vo svojom vedeckom prostredí, ktorých múdrosť a skúsenosti majú potenciál obohatiť široké publikum. Ich schopnosti môžu povzbudiť najmä mladých kolegov a doktorandov na začiatku ich profesionálnej kariéry. Práve preto by mala byť táto konferencia adresovaná aj im a poskytnúť im niečo ako prvú bránu k vedeckým prezentáciám, besedám, diskusiám. Sme veľmi radi, že počas viac ako 20-ročnej histórie konferencie mnohí dnes už veľmi úspešní kolegovia mali svoje prvé verejné alebo vedecké vystúpenie práve na Konferencii košických matematikov.

Teší nás, že tento rok naše pozvanie predniesť prednášku prijali: doc. RNDr. Zuzana Chladná, Dr. rer. soc. oec., doc. Mgr. Ján Karabáš, PhD., doc. RNDr. Daniel Klein, PhD., RNDr. Jana Krajčiová, PhD., prof. RNDr. Michal Krížek, DrSc., doc. RNDr. Edita Máčajová, PhD., Mgr. Jan Širůček, Ph.D. a Mgr. Martina Tischlerová, čo je príslubom kvalitných konferenčných prezentácií.

Dovoľte nám v mene organizačného výboru konferencie vyjadriť nádej, že sa vám táto konferencia bude páčiť a že splní všetky svoje ciele.

Editori: Ján Buša
Erika Fecková Škrabuláková
Andrea Feňovčíková

Editorial

Dear participants of the Conference of Košice Mathematicians and other readers!

The events of the last few years marked by the pandemic forced us to live in isolation for a long time. It was impossible to meet each-other face to face, organize mass events... In the circle of restrictions, it was difficult to make plans, to predict future events, to estimate possible scenarios of their development and to react flexibly to them. Constantly changing restrictions forced us to postpone and reschedule the Conference of Košice Mathematicians again and again, which led to the fact that the twentieth jubilee conference organized in year 2019 is followed by the next one only in year 2023.

The rough months and years have shown us a good mirror of our society and highlighted the importance of scientific research, interpersonal contacts, as well as the links between research and practice. The need for socialization and maintaining social contacts resonates even more in society today, as social connection not only brings us pleasure, but also affects our long-term health. Thanks to online platforms, we had the opportunity to stay in touch during those hard months, but these platforms could not replace face-to-face meetings. From our own experience we know that even scientific research is more fruitful in the case of direct contact. For these reasons, we especially appreciate, dear conference participants, that we can welcome you at the Conference of Košice Mathematicians in person.

The idea of organizing a conference of this type was established at the Union of Slovak Mathematicians and Physicists, branch Košice, at late nineties of the last century. The goal was to provide a platform for meetings of people professionally engaged in mathematics living in the east of Slovakia – for teachers, scientists, applied mathematicians... The aim was to establish a regular forum, where they can discuss with other colleagues, share their joys and worries related to the work; look for solutions of pedagogical, didactical and scientific problems. From the very beginning there was an idea of a serious conference establishment, where high-quality content would be guaranteed especially via invited lectures. Therefore, the invited lecturers were emerging quality personalities, well-known in their scientific environment and whose wisdom and experience would enrich the wide audience. Their skills can enhance especially young colleagues and PhD students at the beginning of their professional carrier. Therefore, this conference should be addressed also to them and provide them something like the first gate to scientific presentations, talks and discussions. We are

very proud that during more than 20 years of conference history many of nowadays very successful colleagues had their first public or scientific performance at the Conference of Košice Mathematicians.

We are happy that this year our invitation to deliver a lecture accepted doc. RNDr. Zuzana Chladná, Dr. rer. soc. oec., doc. Mgr. Ján Karabáš, PhD., doc. RNDr. Daniel Klein, PhD., RNDr. Jana Krajčiová, PhD., prof. RNDr. Michal Krížek, DrSc., doc. RNDr. Edita Máčajová, PhD., Mgr. Jan Širůček, Ph.D. and Mgr. Martina Tischlerová, what is the promise of high-quality conference talks.

On behalf of the organizing committee of the conference, let us express the hope that you will enjoy this conference and that it will fulfill all its aims.

Editors: Ján Buša
Erika Fecková Škrabuláková
Andrea Feňovčíková

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Invited lectures

My personal journey through the mathematical epidemiology

Zuzana Chladná

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In this talk I would like to introduce some interesting topics in mathematical epidemiology which I was working on. We start with basic models and their properties. Further, we discuss one of the main issues in the mathematical epidemiology: how to control the epidemic spread. The modelling challenges connected with two intervention strategies, vaccination and quarantine, will be presented. We show an example when implementing an intervention measure results in a periodic solution, i.e., in a repetitive need to implement the intervention. At the end of the presentation we introduce a game theory model and explain why the herd immunity cannot be reached solely by voluntary vaccination.

Acknowledgement. The presented work was partly supported by the Slovak Scientific Grant Agency VEGA 1/0755/22.

On the relationships in the class of highly symmetric maps

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A cellular graph embedding, a map, is considered as *highly symmetric*, if its group of automorphisms acts transitively on arcs, edges, or vertices of its underlying graph. The regular maps are the prominent example of highly symmetric maps, recognised in several ancient cultures (Plato, ~350 BC). “Less symmetric” maps also attracted people for millenia; they appear e.g., in works of Archimedes (~200 BC), islamic architects (10th–13th century), Johannes Kepler (1619), or M. C. Escher. Thanks to development of algebraic geometry in late 19th century we understood that the study of seemingly geometric problems using algebraic methods give us tools to grasp (at least partially) fundamental problems in number theory, combinatorics, abstract algebra, geometry, or topology.

In the lecture we will describe the relationship between regular maps and vertex-transitive, and edge-transitive maps using algebraic methods. We will show the algebraic counterparts of well known (and well understood) geometric surgeries used to realise a symmetric map from another. Moreover, we will show that using algebra one can generalise from spherical or toroidal map to maps of any genus. Further, the use of algebraic methods allows us not only construct some maps from other, but leads us to deeper questions on symmetries of surfaces. We will show that methods discovered in study of highly symmetric maps can be used in characterisation and classification of Riemann surfaces. This knowledge is widely used in contemporary cryptography, classification of simple groups, or even in quantum physics.

Acknowledgement. The talk is based on the joint work with Roman Nedela and Mária Skyvová. The works were partially supported by the grants APVV-19-0308, VEGA 2/0078/20, and GACR 20-15576S.

Estimation and testing in multivariate linear models

Daniel Klein

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Experiment is one of the main tool of scientific research and it typically results in a set of measured or observed data. There are two main problems to solve while analysing data – to determine the values of unknown parameters, and to test the hypotheses about the values of unknown parameters. The talk will be focused on the problems of estimation and testing of unknown parameters in multivariate linear models – in the growth curve model and its generalized versions as well as in two-level multivariate linear model. The estimators of unknown parameters under some specific conditions, especially under assumption of special covariance matrix structure, as well as tests of covariance structure will be presented.

A teacher of mathematics: Experiences & failures

Jana Krajčiová

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In the process of educating, teachers should account for the fact that students will make mistakes and use them as an opportunity to help students grow. However, teachers are not flawless either. Can a mistake in teaching mathematics be used to better future learning? If so, how would it translate to different classroom settings such as further mathematics lesson or a focus group with higher ability pupils?

The magic of numbers: From great discoveries in number theory to applications

Michal Křížek

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The French mathematician Pierre de Fermat (1601–1665) is considered to be the founder of modern number theory. He became most well known for his pioneering work in the area of prime numbers. His work has been attracting the attention of both amateurs and professional mathematicians for almost 400 years.

The purpose of this lecture is to provide an overview of many interesting properties of natural numbers (Fermat primes, Mersenne primes, etc.) and to demonstrate their numerous appearances and applications in areas such as astronomy, geometry, mechanics, cryptography, theory of chaos, graph theory, image processing, computer tomography, and theory of fractals. In particular, we will present the main ideas of error-detecting and error-correcting codes, hashing functions, generators of pseudorandom numbers, digital signatures, the RSA method based on large prime numbers, JPG compression, and so on. Most of the material has been taken from the monographs [1] and [2].

Acknowledgement. The present work was supported by the Czech Academy of Sciences (RVO 67985840).

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The Fulkerson conjecture and the cycle double cover conjecture

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The Fulkerson conjecture and the cycle double cover conjecture belong to the most prominent open problems in graph theory. Both these conjectures assert statements about double covers. The Fulkerson conjecture suggests that the edges of every bridgeless cubic graph can be covered with six perfect matchings in such a way that each edge belongs to exactly two of them. The origin of this conjecture lies in mathematical programming and the conjecture itself has close connections to configurations of points and lines in the projective space. The cycle double cover conjecture asserts that the edges of any bridgeless cubic graph can be doubly covered with cycles. Despite the fact that both these conjectures were formulated half a century ago, they have been verified only for several explicitly defined families of graphs. During the talk we will present history and the current state of these conjectures as well as their connections to other important problems in graph theory.

The use of “mathematical” statistics in psychology: Our current crisis

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Nomothetic research paradigms across psychological disciplines share two assumptions: the first is the belief in the measurability of psychological phenomena and the second is the belief that relationships between psychologically measured phenomena can be analyzed by means of mathematical statistics. The assumptions in question were formulated in the first half of the 20th century by researchers following the work of Francis Galton and Carl Pearson, and are still taken in the mainstream of psychological research as unwavering facts that do not need to be reflected in any practical way.

This element of belief, held for decades in the practice of empirical research, has resulted in the current situation in several fields of psychological research, which is briefly referred to as the “replication crisis”. Currently, the validity of a number of mainly experimental findings that have failed to be rigorously or freely replicated is gradually being questioned. It is very likely that the fault lies not in any real change in the observed regularities, but in the lack of discipline in following experimental protocols and especially in data analysis. Data analysis misconduct can have two causes: simple ignorance and reliance on long-established procedures whose shortcomings are not reflected by the mainstream, or, on the contrary, more or less subtle manipulation of the process and results of the analysis in the hope that fraud will not be detected in the peer review process.

This talk discusses the possibilities and challenges posed by traditional models of training social science researchers in statistical data analysis and asks what didactic practices might lead out of the current crisis linking inappropriate training with a lack of good manners. Specific examples are misconceptions of the most widely used statistical concepts in psychology: correlation, reliability, and statistical significance.

A perspective on teaching secondary school mathematics

Martina Tischlerová

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The objective of teaching mathematics is to develop students' abilities to apply mathematics and mathematical thinking in their future lives. School is a place that should help promote students' positive attitude towards mathematics and its positive perception. Children are endowed with natural curiosity. When enrolling in a new school, they are full of expectations – they seek further knowledge, they are motivated to learn and find out something new. It is important to be able to actively and creatively employ this curiosity and motivation of children. In teaching mathematics, it is necessary not only to pass on to students the ready-made knowledge and instructions on how to calculate something. It is absolutely crucial to provide sufficient and ample space for students to research, investigate and explore so that they, under the guidance of their teacher, are able to discover new skills, to develop relational thinking. It is essential to lead students to ask questions as to why this is the way to calculate something rather than merely teach them how to calculate it.

The question, however, is whether the current conditions in our schools are conducive to accomplishing these goals. The reality is that a rising number of students find mathematics difficult. Correspondingly, the number of students interested in selecting mathematics as one of the subjects in which to take their secondary-school-leaving exam (“maturita”) has been declining recently. In my talk, I consider and point out the potential reasons behind this trend as seen from the perspective of a secondary school maths teacher. I will illustrate the everyday reality and suggest improvements which would lead, first, to meeting the students' expectations and also, second, to help students live up to the expectations placed on them by others.

Conference contributions

On completeness and topologizability of countable semigroups

Serhii Bardyla

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In this talk we discuss a connection between categorical closedness and topologizability of semigroups. For a class T_1S of T_1 topological semigroups we show that a countable semigroup X with finite-to-one shifts is injectively T_1S -closed if and only if X is T_1S -nontopologizable in the sense that every T_1 semigroup topology on X is discrete. Moreover, a countable cancellative semigroup X is absolutely T_1S -closed if and only if every homomorphic image of X is T_1S -nontopologizable. Also, we discuss a notion of a polybounded semigroup. It is proved that a countable semigroup X with finite-to-one shifts is polybounded if and only if X is T_1S -closed if and only if X is T_zS -closed, where T_zS is a class of Tikhonov zero-dimensional topological semigroups. We show that polyboundedness provides an automatic continuity of the inversion in T_1 paratopological groups and prove that every cancellative polybounded semigroup is a group. The aforementioned results were obtained in collaboration with Prof. Banach and published in [1].

Acknowledgement. The present work was supported by the Slovak Research and Development Agency under the Contract no. APVV-20-0045.

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The computation formula for the generalized Choquet integral

**Stanislav Basarik, Jana Borzová,
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Starting with the works of Vitali [3] and Choquet [2], the development of nonadditive measures and integrals began, and it continues to these days. During this epoch, perhaps the most famous type of nonadditive integral the Choquet integral was defined. It is the improper Riemann integral of the survival function, which in the discrete case can be expressed by the formula

$$\text{Ch}(\mathbf{x}, \mu) := \sum_{i=0}^{n-1} (x_{(i+1)} - x_{(i)}) \cdot \mu(E_{(i+1)}), \quad (1)$$

where $[n] := \{1, 2, \dots, n\}$, $n \geq 1$, $n \in \mathbb{N}$, is a basic set, $\mathbf{x} = (x_1, \dots, x_n) \in [0, \infty)^{[n]}$ is a vector, $\mu: 2^{[n]} \rightarrow [0, \infty)$ is a monotone measure, and (\cdot) is a permutation such that $0 = x_{(0)} \leq x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$, $E_{(i)} = \{(i), \dots, (n)\}$ for $i \in [n]$.

There are several generalizations of the Choquet integral, one of which was considered by Boczek et al. in [1]. They rewrote the survival function as follows

$$\mu(\{i \in [n] : x_i > \alpha\}) = \min \{ \mu(E^c) : \max_{i \in E} x_i \leq \alpha, E \in 2^{[n]} \}$$

for any $\alpha \in [0, \infty)$. Replacing the maximum by the conditional aggregation operator and the power set by its subset we get the generalized survival function

$$\mu_{\mathcal{A}}(\mathbf{x}, \alpha) := \min \{ \mu(E^c) : \mathbf{A}(\mathbf{x}|E) \leq \alpha, E \in \mathcal{E} \}, \quad \alpha \in [0, \infty),$$

with $\mathcal{A} = \{ \mathbf{A}(\cdot|E) : E \in \mathcal{E} \subseteq 2^{[n]} \}$ being a family of conditional aggregation operators. Improper Riemann integral of generalized survival function is the generalized Choquet integral

$$\text{C}_{\mathcal{A}}(\mathbf{x}, \mu) := \int_0^{\infty} \mu_{\mathcal{A}}(\mathbf{x}, \alpha) \, d\alpha.$$

It is a new concept and there is no known formula for its calculation analogous to formula (1) in discrete space. In this contribution, we point out the approaches or constructions with the help of which many calculation formulas for the generalized Choquet integral can be derived. The deriving of formulas we supplement with graphic visualizations and pseudo-algorithms.

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Utilization of active learning elements in the selected course

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College students generally have some concerns about mastering an Introductory Statistics course. Several scientific articles indicate that methodologies in which the student is an active element in the educational process contribute to his greater participation in the teaching activities and to better results in the overall assessment.

The proposal aims to change the Statistics course, which should combine the traditional form of teaching and the addition of active learning elements. These elements involved in teaching contribute to a better understanding of statistical methods and the development of the soft skills of students. The

proposed changes are also based on an analysis of risks and opportunities. The initial analysis included the previous experience of the authors of face-to-face teaching, as well as online teaching during the COVID-19 pandemic, the current requirements of employers, and the results of the satisfaction survey of course graduates. The presented redesign of the Statistics course tries to take into account the results of these analyses as much as possible, as well as the relevant requirements of all interested parties.

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Comparison of teaching methods in the context of the achieved study results of the selected courses

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Statistical literacy and statistical reasoning are an important part of teaching at a technically oriented university. Therefore, active control of statistical methods should be an integral part of the knowledge and skills of managers at different levels in a manufacturing company. It is necessary to take it into account during the professional training of graduates for practice. For current graduates to find good employment and be comprehensively prepared for the demands of the labor market, it is also advisable to change the traditional teaching method, which mainly emphasized the development of hard skills. It is, therefore, necessary to intensively incorporate new methods supporting the development of soft skills into the teaching process, precisely in view of the constantly changing conditions in the labor market.

As a part of the initial analysis, a comparison of the continuous and final evaluations of graduates of subsequent courses (Mathematics 1 and Statistics) was processed. Subsequently, the results of a questionnaire survey among graduates of the Statistics course were processed. This survey was aimed at evaluating the content of the course as well as the implemented

educational activities. One of the goals of the survey was to identify those soft skills, the development of which the students themselves consider important for them. After evaluating the results of the survey, analyzing the experience of educators gained during the twelve years of teaching the Statistics course, as well as the current requirements of employers for university graduates, the team of educators decided to redesign this course.

Acknowledgement. This research was funded by the following grants: VEGA 1/0264/21, KEGA 005TUKE – 4/2022, KEGA 040TUKE – 4/2021, APVV-21-0195, and APVV-18-0526.

Around Cichoń's diagram

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The aim of this talk is to introduce Cichoń's diagram Fig. 1 which is one of the most important diagrams in set theory. We focus to explain some of their basic results and mention some of their recent consistency theorems.

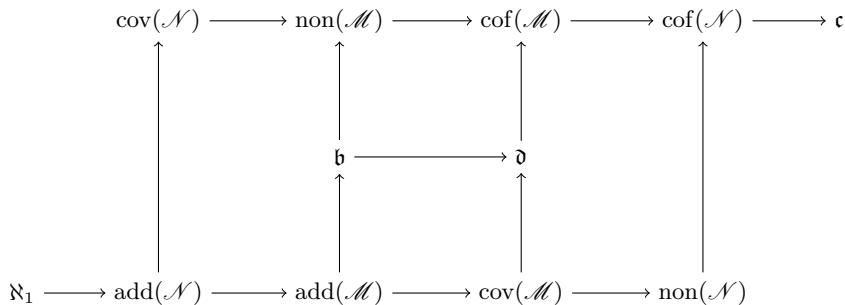


Figure 1: Cichoń's diagram.

Acknowledgement. The present work was supported by the Slovak Research and Development Agency under Contract No. APVV-20-0045 and by Pavol Jozef Šafárik University at a postdoctoral position.

About a platform that has been advancing scientific research for more than twenty years

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Several decades of intensive mathematical research in Košice and its surroundings have brought fruit in the form of many new findings in the fields of vector measures, ordered algebraic structures, set theory, geometry, theory of differential equations, combinatorics, basics of mathematics and others. However, this knowledge from various areas of mathematics was not created in isolation, but on the basis of quality mathematical education, and that at all its levels – starting with the primary one.

A basic prerequisite for rapid progress in mathematical research is intensive communication, sharing of knowledge and information about new scientific findings. This induces the need for a high-quality scientific-pedagogical forum where these presentations could be discussed.

For mathematicians from Košice and the surrounding area, the Conference of Košice Mathematicians has become such a forum, where scientists, applied mathematicians and teachers of mathematics share, seek and find solutions to scientific, didactic and educational problems. In this contribution, we will present its more than twenty-year history.

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On the use of the principal component analysis in humanities and technical disciplines

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The Principal Component Analysis is not rare in the conditions of multivariate statistics, however, it is not much used when dealing with humanities and technical disciplines. In this contribution we want to present several examples of the use of this powerful mathematical tool precisely in the above named areas. We discuss and compare the advantages and disadvantages of the traditional questionnaire survey and the approach based on points when analyzing data (that can be strongly affected by subjective perception of responders or researchers) with the approach based on the Principal Component Analysis. Our objective is to demonstrate that the Principal Component Analysis can be used in many diverse areas — from geoinformatics through the analysis of land cover changes up to evaluating the quality of life, especially when it is necessary to evaluate a significant number of variables and select factors with the highest impact.

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Teacher's knowledge of mathematics content in the topic linear function

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The researchers were looking for a model that could describe mathematics instruction or teacher's knowledge of mathematics content. As a tool for this work, the model would serve to improve the quality of mathematics instruction. The model we chose for our work was the Model Mathematics Teacher's Specialized Knowledge (MTSK), which was developed by a group of Spanish researchers led by Carrillo. We will describe this model in more detail. We then focus on one of its domains, namely, knowledge of the topic (KoT), which deals with the teacher's understanding of the content of a mathematical topic. We will link this part of the model to the topic of linear functions for teaching in primary schools. We have developed a CoT for this purpose. Using the Codex, we will conduct a qualitative analysis of teacher's preparation for teaching linear function in primary school. The aim of the Codex is to consistently describe teacher's knowledge of the content of the linear function topic. Our research revealed gaps in content knowledge in the linear function topic, or we found parts of the curriculum that were not included or were incorrect. We conclude by pointing out that the discovery of gaps in teacher's knowledge about the content of the topic can serve as a springboard for future teacher's education. Because we can use the model to describe content knowledge in the linear function topic, we can identify where teachers have gaps. Based on this, we can, for example, develop a training program for teachers or students studying to be future mathematics teachers.

A mathematical model of the organ sound generated by a collection of pipes of constant measure

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Dedication. Dedicated to prof. Enrique Ramírez de Arellano, mathematician and collector of Mexican folk music and to MUDr. Karol Wurm, my organ teacher.

We worked together with Andrej Štafura from Institute of Materials and Machine Mechanics SAS on the project *Wooden organ dispositions of historical organ positives in Slovakia* in previous years. Now we are preparing a monograph from the results we obtained within the project called *Influence of materials on acoustic properties of historical single-manual pipe organs in Slovakia*.

I will deal with the mathematical part of the upcoming publication in my contribution. I will mention several challenges too. One of them is connected with fullerenes C 84 and C 60, see Fig. 2, since Pythagorean circle is the conjugation of 84 fifths or 60 quarters.

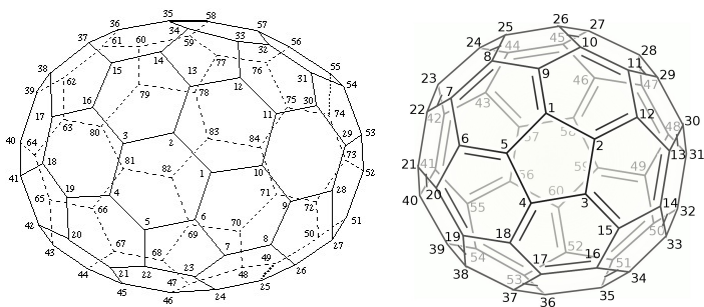


Figure 2: Fullerene C 84 and fullerene C 60.

Keywords. Algebras over field \mathbb{R} , Fourier series, generalized complex numbers, invertible elements, measures of organ pipes, octave equivalence, operations derived from 12-TET and Webern – Fechner’s psycho-physical law, ordered Hilbert algebra, Pythagorean comma, skew circulant matrices.

Acknowledgement. Thanks to Małgorzata Jastrzebska from University Siedlce in Poland for useful mathematical discussions. The work was supported by the grant VEGA 2/0134/23.

On the structure of the space of triads

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In investigating the structure of chords in an equal tempered tone system, some techniques of discrete mathematics can be used, namely group theory and polyhedral theory. The use of dihedral group of order 24 is known in the construction of the so-called T/I-group (with T – translation and I – inversion) and PLR-group (with operations P – paralel, L – leading tone exchange and R – relative) both acting on major and minor chords in the standard 12-tone equal tempered tone system E_{12} of Western music. We can also represent this group graphically, see Fig. 3.

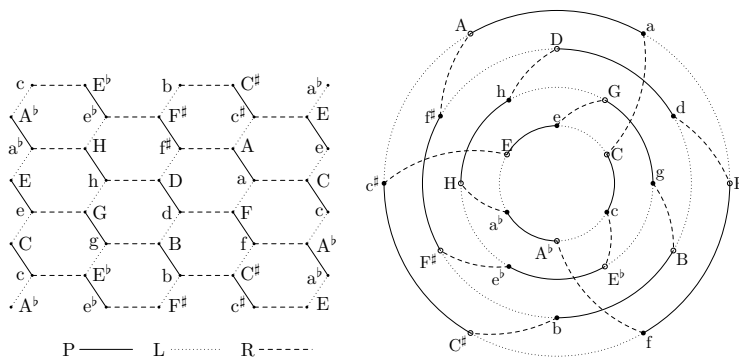


Figure 3: Musical movement on the torus generated by the PLR-group.

This approach allows musical composition techniques to be seen on the clock (with the underlying structure of \mathbb{Z}_{12}). Interesting results are obtained when studying the structure of the space of triads $C(n_1, n_2, n_3)$ (not necessarily major and minor) in the general N -tone equal tempered tone system E_N with the number of tones $N = n_1 + n_2 + n_3$ within one octave. This space can be modeled by a simplicial complex. Computing its Euler characteristic

we can describe the topological structure of the space $C(n_1, n_2, n_3)$. As a consequence we get that the structure of the space $C(3, 4, 5)$ of all major and minor triads in E_{12} is a well known *torus*.

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Variations on Olivier's theorem

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Each student of mathematics encounters the problem of convergence of infinite series of nonnegative real numbers. Standard approach to determining whether such series is convergent or divergent is based on the use of convergence criteria/tests, e.g., d'Alembert's criterion or Cauchy's criterion, see [3]. But, do you remember the following one?

Theorem 1 *If $\lim_{n \rightarrow \infty} na_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent¹. Conversely, if $\lim_{n \rightarrow \infty} na_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ is divergent.*

Theorem 1 was formulated in 1827 by unknown mathematician Louis Olivier [2], but this convergence test (although it looks like a powerful tool) is not taught in the standard courses of mathematical analysis. The reason is the immediate reaction article [1] of Henrik Abel showing that only the necessary condition is true.

Theorem 2 (Olivier-Abel-Pringsheim) *If $(a_n)_1^{\infty}$ is a nonincreasing sequence of nonnegative numbers and $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n \rightarrow \infty} na_n = 0$.*

¹In the original Olivier's paper a series $\sum_{n=1}^{\infty} a_n$ is meant to be convergent, if: (i) $(a_n)_1^{\infty}$ is a decreasing sequence of nonnegative real numbers, (ii) $\lim_{n \rightarrow \infty} (a_{n+1} + a_{n+2} + \dots) = 0$.

Besides talking about historical background on Theorems 1 and 2, we will discuss its generalizations in different directions, e.g., weakening the monotonicity, or the nonnegativity of the sequence. We also show a characterization of infinite series satisfying the statement of Theorem 1.

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Cardinal invariants of meager ideals related to P-like properties

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Given two ideals \mathcal{I} , \mathcal{J} on the set X we say that \mathcal{I} is a $P(\mathcal{J})$ -ideal (a weak $P(\mathcal{J})$ -ideal) if for any countable family $\{I_n : n \in \omega\}$ of elements of \mathcal{I} there is $I' \in \mathcal{I}^*$ ($I' \in \mathcal{I}^+$) such that $I_n \cap I' \in \mathcal{J}$ for all $n \in \omega$. The $P(\mathcal{J})$ -property was introduced by Mačaj and Sleziak in [1] and later applied in various works.

In this talk, based on [2], we study two cardinal characteristics of ideals associated with the above-mentioned P-like properties, namely $\text{cof}^{\mathcal{I}}$ and cov^+ . We shall concentrate on meager ideals on ω , i.e., ideals of the first Baire category with respect to the standard Cantor topology on $\mathcal{P}(\omega)$. In particular, we shall consider some well-known critical ideals having nice representations on $\omega \times \omega$. In addition to some existence results, we provide values of $\text{cof}^{\mathcal{I}}(\mathcal{J})$ for every reasonable pair \mathcal{I} , \mathcal{J} of the aforementioned

critical ideals on $\omega \times \omega$. We show that each of these values is either very small, i.e., 1, or very large, i.e., as large as some base of \mathcal{I} . However, $\text{cof } \mathcal{I}$ does not behave this way in general (except e.g., under CH).

Moreover, using cov^+ -number we obtain a simple way of proving strict inequalities in Katětov order $\text{Fin} <_K \langle \mathcal{A} \rangle <_K \text{Fin} \times \text{Fin}$ for ideals generated by any MAD family \mathcal{A} .

Acknowledgement. This work was supported by the Slovak Research and Development Agency under the Contract no. APVV-20-0045.

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A winning strategy for cutting corners

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Cutting Corners is a paper-and-pencil game for two players. It was introduced in the first edition of *A Gamut of Games* [3] in 1969, one of the most influential books by Sid Sackson, a professional game inventor, collector and columnist.

Fifty years later, the game was mentioned in an inspiring paper *Mathematical Treasures from Sid Sackson* [1] along with three more Sackson's games as an example of an old game without known winning strategy. Jim Henle, the author of this article, argues that Sackson created games that pose interesting mathematical problems.

In this talk, we will take a closer look into Cutting Corners and some of its modifications. We will focus on the winning strategy for this game discovered in 2022 using a program written in Mathematica. The program itself will be introduced as well. A more detailed description of the strategy can be found in [2].

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Mathematical modelling of an internet of things technology

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We live in the age of the 4th Industrial Revolution. The leading technologies of this revolution are Cloud Computing, Big Data, and the Internet of Things (IoT). Our work describes the mathematical and data model of such IoT technology. The vast majority of such technologies are characterized by the fact that we collect data using sensors evaluated using the Internet. Our project MOVIR also implements such an IoT technology. With the help of our project, we are developing an autonomous electronic area or space protection system. One of the areas of application of the plan is to monitor the quarantine zones created during epidemics. The idea to create a system of mobile sensory units for defending quarantine zones was born during one of the phases of the COVID-19 outbreak in 2020. A network of mobile sensor units defines the quarantine zone. Our work deals with the mathematical description of such quarantine zones. Our article, in which the concepts and algorithms included in the abstract are explained in more detail, can be found here [2]. One of our articles [1] related to air traffic

control gave the idea of modelling the protected space. Here we define the area of interest using significant points. Points are given using GPS coordinates. With the help of a spatial coordinate system, these significant points and a projection, we define a coordinate system in which we can define and model our protected area. Here, a digital terrain model, where significant points are located, is required as input data. This digital (raster) model is defined using a matrix whose elements indicate the height of points in space. The row and column indices of the matrix determine the details of the area. We can use several height layers to describe different obstacles. Our data model contains, in addition to the description of the digital spatial model, the specification of the starting point of our coordinate system and the definition of the x and y -axis based on significant points. In the future, using the data model, we would like to develop a simple geographic information system based on which it will be possible to create a sensor network of the protected area.

Keywords. Earth centered inertial, geodetic coordinate systems, digital model of a terrain, digital model of the protected area, network data model, network of mobile sensory units.

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Mini tutorial: Active learning in engineering education in mathematics, part I

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Students do not come to a technical university to study mathematics as such. However, mathematics is an integral part of their studies and in many cases also of their further professional focus. From our point of view, it is important that students acquire the right skills in formulating their thoughts, asking questions, and finding answers to these questions using mathematics. However, the inquiry remains how to manage the educational process so that the educational goals are fulfilled, and the knowledge, skills, and competences of students are sustainable and applicable in solving engineering problems. The first part of this tutorial focuses on adherence to the basic principles of active learning, where the attention is on the student's activity, and the teacher is in the role of mentor, advisor, coach. Part of this section will introduce our experience with the implementation of flipped learning, SCRUM, cooperative learning, collaborative learning, problem based learning, and project based learning in the teaching of subjects with mathematical content at the Faculty of Mining, Ecology, Process Control and Geotechnologies of the Technical University of Košice.

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Mini tutorial: Active learning in engineering education in mathematics, part II

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This part of the tutorial is dedicated to the modernization of teaching materials for the teaching of subjects with mathematical content at the Faculty of Mining, Ecology, Process Control and Geotechnologies of the Technical University of Košice. We will explain individual methods of its design, its usage directly in the teaching process, as well as our experience with the effectiveness of applying various tools we have created. We will present the possibilities of generating static online collections using Google's free service for creating private blogs (Blogger). Such a collection can be expanded with animations in the L^AT_EX system using the L^AT_EX Animate package with output to interactive files in PDF format, as well as illustrations of the procedure for solving selected sample tasks in JavaScript with output to interactive web pages. Another form of teaching materials are dynamic materials prepared in the MATLAB Live Editor environment (LiveScripts) intended for experimentation with mathematical concepts and objects as part of teaching both in school and at home. Verification of knowledge at several levels is an integral part of the educational process. To do this, we have created an extensive generator of unique tests in the MATLAB GUI environment, along with the ability to create correct solutions to quickly check the correctness of the results obtained by students. Each student receives an individual set of tasks of comparable complexity.

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Mini tutorial: Active learning in engineering education in mathematics, part III

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In this part of the tutorial, attention will be paid to the Learning Management System (LMS) Moodle, a description of this system, an analysis of its advantages and disadvantages, a way how it is handled both from the perspective of the teacher and the student. The development of standard activities in LMS Moodle related to the creation of tests to check the knowledge, skills, and competences of students during the teaching process in subjects with mathematical content will be presented. Furthermore, we will focus on online education during the COVID-19 pandemic at the Faculty of Mining, Ecology, Process Control and Geotechnologies of the Technical University of Košice. We will describe our experience with managing all phases of the educational process in the online environment, as well as the impact of this sudden change on all participants in the educational process.

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Mini tutorial: Active learning in engineering education in mathematics, part IV

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The online age brought a great boom in information technologies and their rapid introduction into all areas of human activity. It would not be right to return completely to the times before this powerful experience, but it is advisable to find the right measure of combining face-to-face teaching and online teaching so that the educational goals can be achieved effectively and the knowledge, skills, and competences of the students are long-term and usable not only in their professional education, but also in their further professional practice. Implementation of our approach means significant changes in the massive (or bulk) delivery of knowledge using available information technologies. The main benefits of the presented system consist in the increased resulting level of knowledge of students along with their satisfaction with the results and the form of their study. The most important changes arising from our approach are the following. First, the study process became distributed in space and in time. Second, it can be piecewise continuous in time, and, since all students can study at their own pace, it runs in multiple individual time scales. The most important change, however, is the shift of the paradigm the educational process from transmissive “teach–learn” to active “study”.

Acknowledgement. This research was funded by the Cultural and Educational Grant Agency MŠVVaŠ SR (KEGA) under grant 040TUKE-4/2021 and by the Slovak Research and Development Agency (APVV) under grant APVV-18-0526.

Standard and generalized level measure and their application in scientometry

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The standard level measure is a key concept in many fields. Apart from integrals, on the basis of the generalized level measure the convergence of a sequences of functions in measure is built. The level measure is also used, for example, in mathematical statistics. In the contribution we deal with the generalization of level measures, see [1, 3]. The key elements of the generalization are the conditional aggregation operators introduced in [2]. This concept is worth to study because it can bring solutions to practical problems, which cannot be solved with a standard level measure.

In our talk we will point out the use of level measures in scientometry. We will deal with the standard level measure and its relation to the generalized level measure. We will formulate the necessary and sufficient conditions for equality and inequalities between the generalized and standard level measures. As a consequence we will get sufficient conditions for equality and inequalities between some standard and generalized nonadditive integrals. Another consequence is the equality of the standard Hirsch index with other citation indices. We will use generalized level measures for calculation various citation indices. We will compare several scientometric indicators on real data [4].

Acknowledgement. This work was supported by the APVV-21-0468 and VEGA 1/0657/22 grants.

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Experimental design as a practical statistical tool

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The current time, when the competition in the market is very strict, requires, in addition to the prescribed quality of the products sold, high production efficiency. Statistics in such an environment is not just a theoretical tool that explores probability and helps to evaluate practical claims. In the form of experimental design, it comes into action as a useful tool for improving the results of industrial processes and for improving the processes themselves. The talk deals with experimental design. Some of its historical milestones are described here and some people who contributed to its development are mentioned. It briefly presents real cases of using the design of experiments in industrial processes with the aim of improving industrial processes and with regard to sustainability, i.e., limited resources for the realization of the experiment and limited resources for the industrial processes.

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Fréchet–Urysohn property, slalom numbers

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We discuss results from two different topics on the borderline of topology and set theory. Our research on the Fréchet–Urysohn property of a space of continuous functions led to a solution of the old problem posed by Gerlits and Nagy [1]. The second part is devoted to cardinal invariants, called slalom numbers, and their connection to topological spaces. Both topics are investigated in collaboration with other researchers.

Acknowledgement. This work was supported by the Slovak Research and Development Agency under Contract no. APVV-20-0045.

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Program 21. Konferencie košických matematikov**Programme
of the 21st Conference of Košice Mathematicians****Štvrtok – Thursday 20. 4. 2023****12⁰⁰ – Registácia účastníkov – Participants Registration****13⁰⁰ – 14⁰⁰ Obed – Lunch****14⁰⁰ – 14⁰⁵ Slávnostné otvorenie konferencie – Conference opening****14⁰⁵ – 14²⁵ Fecková Škrabuľáková (ÚRIVP FBERG TUKE): *About a platform that has been advancing scientific research for more than twenty years*****14²⁵ – 14⁴⁵ Frančáková (ÚRIVP FBERG TUKE): *On the use of the principal component analysis in humanities and technical disciplines*****14⁴⁵ – 15⁰⁵ Michalík (KDM MFF UK Praha): *A winning strategy for cutting corners*****15⁰⁵ – 15³⁵ Občerstvenie – Coffee-break****15³⁵ – 15⁵⁵ Gönciová (ÚM PF UPJŠ): *Teacher's knowledge of mathematics content in the topic linear function*****15⁵⁵ – 16¹⁵ Stehlíková (ÚRIVP FBERG TUKE): *Experimental design as a practical statistical tool*****16¹⁵ – 16³⁵ Haluška (MÚ SAV): *A mathematical model of the organ sound generated by a collection of pipes of constant measure*****16⁵⁰ – 18²⁰ Prekvapenie konferencie – Conference surprise****18³⁰ – Večera – Dinner**

Piatok – Friday 21. 4. 2023

7⁰⁰ – 7⁵⁰ **Raňajky – Breakfast**

7⁵⁰ – 8¹⁰ Bednárová (ÚRIVP FBERG TUKE): *Utilization of active learning elements in the selected course*

8¹⁰ – 8³⁰ Bogdanovská (ÚRIVP FBERG TUKE): *Comparison of teaching methods in the context of the achieved study results of the selected courses*

8³⁰ – 8⁵⁰ Kleinová (ÚM PF UPJŠ): *Variations on Olivier's theorem*

8⁵⁵ – 9¹⁵ Marton (ÚM PF UPJŠ): *Cardinal invariants of meager ideals related to P -like properties*

9¹⁵ – 9³⁵ Basarik (ÚM PF UPJŠ): *The computation formula for the generalized Choquet integral*

9³⁵ – 9⁵⁵ Slovinská (ÚM PF UPJŠ): *Standard and generalized level measure and their application in scientometry*

9⁵⁵ – 10¹⁵ **Občerstvenie – Coffee-break**

10¹⁵ – 11⁰⁵ Chladná (KAMŠ FMFI UK Bratislava): *My personal journey through the mathematical epidemiology*

11¹⁰ – 12⁰⁰ Křížek (MÚ AV ČR Praha): *The magic of numbers: From great discoveries in number theory to applications*

12⁰⁰ – 12⁴⁵ **Obed – Lunch**

12⁴⁵ – 13³⁵ Karabáš (KI FPV UMB Banská Bystrica): *On the relationships in the class of highly symmetric maps*

13³⁵ – 13⁵⁵ Hurajová (ÚM PF UPJŠ): *On the structure of the space of triads*

14⁰⁰ – 14⁵⁰ Máčajová (KI FMFI UK Bratislava): *The Fulkerson conjecture and the cycle double cover conjecture*

14⁵⁰ – 15¹⁰ **Občerstvenie – Coffee-break**

15¹⁰ – 16⁰⁰ Tischlerová (Gymnázium Poštová Košice): *A perspective on teaching secondary school mathematics*

- 16⁰⁵ – 16⁵⁵ Krajčiová (Gymnázium Alejová Košice): *A Teacher of mathematics: Experiences & failures*
- 17⁰⁰ – 17²⁰ Pócssová (ÚRIVP FBERG TUKE): *Mini tutorial: Active learning in engineering education in mathematics, part I*
- 17²⁰ – 17⁴⁰ Mojžišová (ÚRIVP FBERG TUKE): *Mini tutorial: Active learning in engineering education in mathematics, part II*
- 17⁴⁰ – 18⁰⁰ Valentová (ÚRIVP FBERG TUKE): *Mini tutorial: Active learning in engineering education in mathematics, part III*
- 18⁰⁰ – 18²⁰ Pócssová (ÚRIVP FBERG TUKE): *Mini tutorial: Active learning in engineering education in mathematics, part IV*
- 18³⁰ – **Večera a spoločenský večer – Dinner & Party**

Sobota – Saturday 22. 4. 2023

- 7⁰⁰ – 7⁵⁰ **Raňajky – Breakfast**
- 7⁵⁰ – 8¹⁰ Cardona (ÚM PF UPJŠ): *Around Cichon's diagram*
- 8¹⁰ – 8³⁰ Šupina (ÚM PF UPJŠ): *Fréchet-Urysohn property, slalom numbers*
- 8³⁰ – 8⁵⁰ Bardyla (ÚM PF UPJŠ): *On completeness and topologizability of countable semigroups*
- 8⁵⁰ – 9¹⁰ Muszka (KLTPLF TUKE): *Mathematical modelling of an internet of things technology*
- 9¹⁵ – 10⁰⁵ Klein (ÚM PF UPJŠ): *Estimation and testing in multivariate linear models*
- 10⁰⁵ – 10²⁵ **Občerstvenie – Coffee-break**
- 10²⁵ – 11¹⁵ Širůček (KP FSS MU Brno): *The use of “mathematical” statistics in psychology: Our current crisis*
- 11¹⁵ – 11²⁰ **Záver konferencie – Conference closing**
- 11²⁰ – **Obed – Lunch**

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