**Jednota slovenských matematikov a fyzikov**
**Pobočka Košice**

**Prírodovedecká fakulta UPJŠ**
**Ústav matematických vied**

**Fakulta baníctva, ekológie, riadenia a geotechnológií TU**
**Ústav riadenia a informatizácie výrobných procesov**

# 20. Konferencia košických matematikov

**Herľany**
**25. – 27. apríla 2019**


ZVÄZ SLOVENSKÝCH
VEDECKOTECHNICKÝCH
SPOLOČNOSTÍ


Konferenciu finančne podporili:

ZSVTS – Zväz slovenských vedeckotechnických spoločností a
SSAKI – Slovenská spoločnosť aplikovanej kybernetiky a informatiky.

Editori: Ján Buša, Erika Fecková Škrabuľáková

ISBN 978-80-553-3293-2

Sadzba programom pdfLaTeX

# Predhovor

Písal sa rok 1998. Bolo krátko po Zimných olympijských hrách v Nagane, v Japonsku. Príroda hrala jarné melódie a upozorňovala na dobré správy visiace vo vzduchu. Údaje poslané sondou Galileo naznačovali, že pod hrubou vrstvou Jupiterovho mesiaca Európa sa nachádza tekutý oceán. NASA vyhlásila, že sonda Clementine na mesačnom orbite objavila v polárnych kráteroch Mesiaca dostatok vody na podporu ľudskej kolónie a stanice dopĺňania raketového paliva. Na univerzite v nemeckom Regensburgu vykonali prvý medicínsky zákrok, v ktorom za pomoci počítača hľadali anatomické pozície nevhodne umiestnených fragmentov kostí v zlomeninách. A v malej dedinke nazývanej Herľany sa okolo 50 výskumníkov stretlo na Konferencii košických matematikov.

Nejako takto by básnik začal knihu o Konferencii košických matematikov s prvými stránkami datovanými do 17.–18. apríla 1998, ktorá sa od toho času koná v Herľanoch každoročne (s výnimkou rokov 2000 a 2004).

Myšlienka zorganizovať konferenciu tohto typu vznikla v košickej pobočke JSMF pod vedením profesora Jendroľa. Cieľom bolo poskytnúť platformu pre stretnutia ľudí profesionálne sa zaoberajúcich matematikou žijúcich na východe Slovenska – pre učiteľov, vedcov, aplikovaných matematikov aj ďalších. Predstava bola vytvoriť pravidelné fórum, na ktorom môžu diskutovať s ostatnými kolegami, podeliť sa o svoje radosti i starosti súvisiace s prácou, hľadať a nachádzať riešenia pedagogických, didaktických i vedeckých problémov. Už prvopočiatočná myšlienka sa týkala serióznej konferencie, ktorej kvalitný obsah bude garantovaný hlavne pozvanými prednáškami. Preto pozvaní prednášajúci mali byť vedecky zrelé osobnosti veľmi známe vo svojom prostredí, ktorých vedomosti a zručnosti majú potenciál obohatiť širokú verejnosť. Ich skúsenosti sú prínosom najmä pre mladých kolegov a PhD študentov na začiatku ich profesionálnej kariéry. Preto táto konferencia má byť cielená i na nich a poskytnúť im niečo ako vstupnú bránu do vedeckých prezentácií, prednášok a diskusií.

Počas rokov mnohí z dnes už veľmi úspešných kolegov absolvovali svoje prvé verejné a vedecké vystúpenia práve na tejto konferencii. Stala sa akýmsi odrazovým mostíkom pre mnohých výskumníkov aj vyučujúcich. A sme na to patrične hrdí. Preto v zmysle tradície prvý deň konferencie je zameraný najmä na mladých kolegov, kým pozvané prednášky sú sústredené do piatkového a sobotňajšieho programu. Neformálne vzťahy sú upevňované počas piatkového spoločenského večera.

Tento rok píšeme už jubilejnú 20. kapitolu Konferencie košických matematikov. Sme radi, že pri tejto príležitosti naše pozvanie prednášať na konferencii prijali: prof. RNDr. dr. Hab. Jan Andres, DSc., prof. RNDr.

Lev Bukovský, DrSc, prof. RNDr. Martin Bača, CSc., prof. RNDr. František Kuřina, CSc., prof. RNDr. Josef Molnár, CSc. a Dr hab. Jakub Przybyło, Ph.D, čo je prísľubom skutočne vysoko kvalitných konferenčných prednášok.

Prajeme si, aby sa všetkým účastníkom akcia páčila a aby Konferencia košických matematikov napĺňala všetky svoje ciele nielen v týchto dňoch, ale aj v mnohých nasledujúcich rokoch. Vychutnajte si ju!

Organizátori: Ján Buša
Jozef Doboš
Erika Fecková Škrabuľáková
Ľubomír Mucha

## Preface

It was in 1998, shortly after the Winter Olympics in Nagano, Japan. The nature played spring melodies and pointed out to some good news hanging in the air. Data sent from the Galileo probe indicated that Jupiter's moon Europa has a liquid ocean under a thick crust of ice. NASA announced that the Clementine probe orbiting the Moon has found enough water in polar craters to support a human colony and rocket fueling station. The first Computer-assisted Bone Segment Navigation was performed at the University of Regensburg in Germany. And in a small village called Herľany round 50 researchers met at the Conference of Košice Mathematicians.

Somehow like this a poet would start a book about Conference of Košice Mathematicians which first pages are dated to April 17–18, 1998 and which was since than organized in Herľany every year (except of 2000 and 2004).

The idea of organizing a conference of this type was established at the Union of Slovak Mathematicians and Physicists, branch Košice, under the leadership of Prof. Jendroľ. The goal was to provide a platform for meetings of people professionally engaged in mathematics living in the east of Slovakia – for teachers, scientists, applied mathematicians, ... The aim was to establish a regular forum where they can discuss with other colleagues, share their joys and worries related to the work; look for solutions of pedagogical, didactical and scientific problems. From the very beginning there was an idea of a serious conference establishment where high-quality content would be guaranteed especially via invited lectures. Therefore, the

invited lecturers were emerging quality personalities, well-known in their scientific environment and whose wisdom and experience would enrich the wide audience. Their skills can enhance especially young colleagues and PhD students at the beginning of their professional carrier. Therefore, this conference should be addressed also to them and provide them something like the first gate to scientific presentations, talks and discussions.

During the years many of nowadays very successful colleagues had their first public or scientific performance at this conference. It has become a high-spring board for a number of researchers and educators. And we are very proud of it. Therefore, according to the tradition, the first day of the conference is devoted mainly to young colleagues, while invited lectures take place on Friday and Saturday. Folksy relations are tightened during the social evening on Friday.

This year we are writing the jubilee 20th chapter of the Conference of Košice Mathematicians. We are happy that by this occasion our invitation to deliver a lecture accepted Prof. RNDr. dr. Hab. Jan Andres, DSc., Prof. RNDr. Lev Bukovský, DrSc, Prof. RNDr. Martin Bača, CSc., Prof. RNDr. František Kuřina, CSc., Prof. RNDr. Josef Molnár, CSc. and Dr hab. Jakub Przybyło, Ph.D., what is the promise of high-quality conference talks.

We wish that all the participants would enjoy the event and that the Conference of Košice Mathematicians will fulfill all its aims not only these days, but also in many forthcoming years. Enjoy it!

Organizers: Ján Buša
Jozef Doboš
Erika Fecková Škrabuľáková
Ľubomír Mucha

# Obsah – Contents

# Invited Lectures

## Sharkovsky Theorem and Differential Equations

**Jan Andres**

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It is well known that the standard version of the celebrated Sharkovsky cycle coexistence theorem [7] from 1964 cannot be applied to scalar ordinary differential equations, satisfying a uniqueness condition. In the lack of uniqueness, the associated Poincaré translation operators along the trajectories of given equations become multivalued. The related multivalued version of the Sharkovsky theorem already applies [2], but with some exceptional absent orbits, which can be exactly described and supported by examples. On the other hand, any nontrivial subharmonic periodic solution of a given differential equation implies [1, 6], rather surprisingly, the coexistence of periodic solutions with all natural multiples of a basic period. Thus, in particular, *Period two implies all periods*, which reminds the title: *Period three implies chaos* of another classical paper [5] by Li and Yorke from 1975.

In our talk, we will explain this phenomenon, jointly with indicating some further possibilities like the consideration of a given differential equation in a (mod 1)-way, which demands to develop a multivalued version [3] of the Block cycle coexistence theorem on the circle [4] from 1981.

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# Local Antimagic Vertex Coloring of Graphs

**Martin Bača**

Department of Applied Mathematics and Informatics,
Technical University, Letná 9, Košice, Slovakia

Let $G = (V, E)$ be a connected graph of order $n$ and size $m$ and let $f : E \to \{1, 2, \ldots, m\}$ be a bijection. For each vertex $u \in V(G)$, the weight $w(u) = \sum_{e \in N(u)} f(e)$, where $N(u)$ is the set of edges incident to $u$. If $w(u) \neq w(v)$ for any two distinct vertices $u$ and $v \in V(G)$, then $f$ is called an *antimagic labeling* of $G$. A graph $G$ is called *antimagic* if $G$ has an antimagic labeling.

The concept of antimagic labeling was introduced by Hartsfield and Ringel [1] and they put forth the following conjectures.

**Conjecture 1** [1] *Every connected graph other than $K_2$ is antimagic.*

**Conjecture 2** [1] *Every tree other than $K_2$ is antimagic.*

Both these conjectures are still open.

A bijection $f : E \to \{1, 2, \ldots, m\}$ is called a *local antimagic labeling* if for any two adjacent vertices $u$ and $v$, $w(u) \neq w(v)$. A graph $G$ is *local antimagic* if $G$ has a local antimagic labeling.

Clearly if $G$ is antimagic, then $G$ is local antimagic.

Hence we have the following conjectures.

**Conjecture 3** [2] *Every connected graph other than $K_2$ is local antimagic.*

**Conjecture 4** [2] *Every tree other than $K_2$ is local antimagic.*

Conjecture 1 implies Conjecture 3 and Conjecture 2 implies Conjecture 4.

Any local antimagic labeling induces a proper vertex coloring of $G$ where the vertex $v$ is assigned the color $w(v)$. Thus, the *local antimagic chromatic number* $\chi_{la}(G)$ is defined to be the minimum number of colors taken over all colorings of $G$ induced by local antimagic labelings of $G$.

Several basic results on this parameter will be present in our talk.

**Acknowledgement.** The present work was supported by VEGA 1/0233/18.

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## Finity and Infinity (in Mathematics)

**Lev Bukovský**

Institute of Mathematics FSc, Pavol Jozef Šafárik University, Jesenná 5, 041 54 Košice, Slovakia

I will briefly recall the history of the concept of infinity in mathematics: from Pythagoras to Cantor, first. Then I will briefly discuss the different philosophical directions in mathematics at the beginning of the twentieth century, which arose as a response to the concept of infinity. Finally, I will show that the assumptions of higher infinity have an unexpected impact on the structure of the real numbers.

# Mathematics as a Part of Culture

**František Kuřina**

University of Hradec Králové, Hradec Králové, Czech Republic

Mathematics is closely connected with the culture of emerging civilization. Arithmetic was developed for the needs of exchange and geometry for the needs of building construction. This development can be illustrated by numerous extant materials. The Pythagorean theorem had been used for thousands of years before Pythagoras, the Cavalieri principle was discovered by Archimedes. I will give more examples in the lecture.

Mathematical culture should be cultivated from the beginning of school education. However, this is often not the case. We can prove it with absurd conventions, teaching mathematics is not connected with developing pupils' thinking, sometimes it is formal. The language of mathematics is sometimes unnatural, the logical aspects are neglected. Problems solved at school are not attractive to pupils. More specifically, I will return to the issue in my speech.

As Eduard Čech, perhaps the greatest Czech mathematician of the twentieth century, said: "... the problem is that mathematics was, is, and will remain a difficult subject. Love for mathematics must be perceived as a significant part of love for work generally".

In my opinion, teaching mathematics should strive to achieve the following goals:

a) To bring all pupils to the level of mathematical literacy needed for life and further study.

b) To develop the mathematical culture of gifted pupils to the first steps of creativity.

c) To cultivate important mental functions (e.g. perception, concentration, thinking, feeling, ...).

d) To cultivate important social functions (e.g. responsibility, diligence, perseverance, criticality, ...).

Will we succeed, with the help of the family and the joint influence of the society, in education every pupil into a good citizen, educated layman or even an expert?

# Mathematical School Terminology Yesterday, Today and Tomorrow

**Josef Molnár**

Department of Algebra and Geometry, Faculty of Science,
Palacký University,
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After the introductory language corner, we will focus on examples of the development of Czech mathematical terminology in particular, we will recall how our ancestors counted and say something about the international and national terminology norm and about the work of the Terminological Commission for School Mathematics of the Union of Czech Mathematicians and Physicists (JČMF).

# Adjacent Vertex Distinguishing Edge Colourings

**Jakub Przybyło and Jakub Kwaśny**

AGH University of Science and Technology, Kraków, Poland

Let $G = (V, E)$ be a simple graph. Consider an edge colouring $c : E \to C$. For a given vertex $v \in V$, by $E(v)$ we denote the set of all edges incident with $v$ in $G$, while the set of colours associated to these under $c$ is denoted as: $S_c(v) = \{c(e) : e \in E(v)\}$. The colouring $c$ is called *adjacent vertex distinguishing* if it is proper (i.e., adjacent edges receive distinct colours) and $S_c(u) \neq S_c(v)$ for every edge $uv \in E$. It exists if only $G$ contains no isolated edges. The least number of colours in $C$ necessary to provide such a colouring is then denoted by $\chi'_a(G)$ and called the *adjacent vertex distinguishing edge chromatic number* of $G$. Obviously, $\chi'_a(G) \geq \chi'(G) \geq \Delta$, where $\Delta$ is the maximum degree of $G$, while it was conjectured [5] that $\chi'_a(G) \leq \Delta + 2$ for every connected graph $G$ of order at least three different from the cycle $C_5$. Hatami [1] proved the postulated upper bound up to an

additive constant by showing that $\chi'_a(G) \leq \Delta + 300$ for every graph $G$ with no isolated edges and with maximum degree $\Delta > 10^{20}$. His proof constitutes a nice exemplary exposition of an application of the famous *probabilistic method* in graph theory. A different approach, based on the newly developed *entropy compression method* was recently successfully applied by Joret and Lochet [2] to obtain a strengthening of the bound above to $\Delta + 19$ (for $\Delta$ large enough).

Suppose now that every edge $e \in E$ is endowed with a list of available colours $L_e$. The *adjacent vertex distinguishing edge choice number* of a graph $G$ (without isolated edges) is defined as the least $k$ so that for every set of lists of size $k$ associated to the edges of $G$ we are able to choose colours from the respective lists to obtain an adjacent vertex distinguishing edge colouring of $G$. We denote it by $\mathrm{ch}'_a(G)$. Analogously as above, $\mathrm{ch}'_a(G) \geq \mathrm{ch}'(G)$, while the best general result on the classical edge choosability implies that $\mathrm{ch}'(G) = \Delta + O(\Delta^{\frac{1}{2}} \log^4 \Delta)$, see [4]. Extending the thesis of this, a four-stage probabilistic argument from [3] grants that $\mathrm{ch}'_a(G) = \Delta + O(\Delta^{\frac{1}{2}} \log^4 \Delta)$ for the class of all graphs without isolated edges. All these problems shall be discussed within the talk, along with a few related concepts (e.g. with distinction of the adjacent sets of colours replaced by the stronger requirement of their mutual non-inclusion).

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# Conference Contributions

## Modelling the Transplant Waiting List

**Katarína Cechlárová, Diana Plačková, Tatiana Baltesová**

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Patients with endstage renal disease have currently two treatment options: dialysis or kidney transplantation. The lack of organs from deceased donors is a world-wide problem, and a willing living donor is often not suitable for medical or immunological reasons. Therefore several countries have started kidney exchange programs, these, however, require larger patient-donor pools to be efficient.

The present work models the situation characteristic for Slovakia. We use Poisson processes to assess the possible evolution of the transplant waiting list and patients' waiting time for an organ, depending on several different parameters of the population and under different policies, including paired kidney donations.

**Acknowledgement.** This work was supported by the Slovak Research and Development Agency under the contract No. APVV-17-0568.

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# Using GeoGebra to Solving Equations and Inequalities

**Jozef Doboš**

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In our contribution, we would like to show how it is possible to use GeoGebra for solving equations and inequalities in school Mathematics.

**Acknowledgement**. The research has been supported by VEGA grant 1/0265/17.

# Mathematical Tool for Suppliers Evaluation

**Erika Fecková Škrabuľáková**

Institute of Control and Informatization of Production Processes, Faculty BERG, Technical University of Košice, Boženy Němcovej 3, 042 00 Košice, Slovakia

Businesses of today are depending on strategic relations between customers and their suppliers. One of the most critical activities of a modern firm is suppliers evaluation and selection. We propose a heuristic for identifying the key supplier of a company (see [1]). In order to develop the suppliers' evaluation tool we use the multicriterial data analysis. We use the principal component analysis as the main statistical instrument here. The processed data were the real data from all realised orders of a chosen company during a period of one year (see [2]) and after their processing the key supplier was identified. As the company is planning to sign an exclusive contract with the identified key supplier, the study helps to close the gap between theoretical work on principal component analysis and actual practice.

**Acknowledgement**. This work was supported by the Slovak Research and Development Agency under the contract No. APVV-14-0892.

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# Student's Value of Study Mathematics and Economics

**Elena Grešová and Erika Fecková Škrabuľáková**

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In presented contribution the issue of connection between practice and teaching, specifically mathematics and economics, is resolved. This objective is perceived from the students' point of view. The questionnaire method is used to identify students' beliefs. The principal statements are formulated in pursuance of analyzing the gathered data and their further collating according to various sets of students. The reasons of recognized findings are examined, as well. This contribution offers the sight on contemporary teaching and attempts to give the proposals for achieving the progress within researched issue. The summary of presented results can be found in [1].

**Acknowledgement.** This work was supported by the Slovak Research and Development Agency under the contract No. APVV-14-0892.

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# Using Card Games in Probability Tasks at Secondary School

**Tadeáš Gavala**

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Jesenná 5, 041 54 Košice, Slovakia

In the talk, formative assessments tools encouraging the investigation and analysis of probability and causal patterns in games with cards are described. The proposed learning activity was tested in one class at secondary school. Prediction card, sequence of tasks in the form of worksheets, and self-assessment card allowed the control of inquiry work and the evaluation of the gained experience. The tasks were focused on investigation of the process and results of the card games and listing the discovered findings together with the justification of the answers. Various forms of the formative assessment were used in the inquiry activity in order to encourage students.

# Beyond the Hirsch Index

**Anton Hovana and Ondrej Hutník**

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Nowadays, the question of measuring quality of scientist's output is still actual and popular. The first approach how to compare scientists was stated by Hirsch in [3], especially for Physics. The index introduced therein has become very famous and wide-spread used in different areas of Science, so it is rightly called the $h$-index. Because of unlimited its domain, it may be used whenever there is need to combine quality and quantity of a researcher represented by a non-negative integers into a single value. Later on, Torra and Narukawa in [8] showed that the $h$-index is a special case of the discrete Sugeno integral with respect to the counting measure. As it can be seen in [6] there are many defects of the $h$-index. Therefore, Mesiar and Gagolewski improved it by *upper* and *lower* 2-$h$-index. There are other indices known

in the literature, see [4] and [5] which are related to the Sugeno and Shilkret integral respectively or measure-modified approach in [7]. Generalization of some previous results can be found in [1]. Further extension of scientometric indices can be done using the concept of integrals based on super level measure. In this concept the integrated function is modified by a certain mapping called a *size* which definition is stated in [2].

**Acknowledgement.** The support of the grants APVV-16-0337 and SK-PL-18-0032 is kindly announced.

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# Meeting Gabor and Choquet Somewhere in an Outer Measure Space

**Ondrej Hutník**

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The 20th century has given the world many outstanding scientists: undoubtedly Dennis Gabor (1900–1979) and Gustave Choquet (1915–2006) being two of them. The first one is considered to be a father of time-frequency analysis being a part of (applied) mathematics originated in the early development of quantum mechanics and in the theoretical foundation of information theory and signal analysis [2]. The second one is known for creating the Choquet theory, the Choquet integral and the theory of capacities [1] — recently understood as basic ingredients of the theory of non-additive measures and integrals.

Non-additive measures do not necessarily satisfy additivity for disjoint finite or countable collections of sets. Lacking additivity for disjoint sets, one cannot expect a useful linear theory of integrals with respect to a non-additive measure. Previously developed theories based on the Choquet integral involves the outer measure of super level sets $\{x : f(x) > ft\}$ for a function $f$. Instead, one can use a more subtly defined quantity to replace the outer measure of a super level set. This new quantity, called a *super level measure* [3], involves predefined averages over the generating sets of the outer measure. The definition reminds the construction of well-know outer measure, but these notions coincide only in some cases. The first example of an outer measure space in which the definition of super level measure does not coincide with the level measure is the upper half-plane in the complex plane where the outer measure is generated by basic sets (the so-called tents). The essentially bounded functions with respect to the outer measure in this upper half-plane are Carleson measures. The main idea behind the scene consists of the factorization of the multilinear form into (Carleson) embeddings, which seems to be an extremely strong tool usable also in other applications of harmonic and time-frequency analysis. In traditional time-frequency analysis, one proves bounds of multilinear forms passing through model sums, where the summation index runs through a discrete set. However, by using the approach of outer measure spaces one

need not pass through a discrete model form, but we could work with the function spaces on a continuum.

We aim to describe basic ingredients of the theory of outer measure spaces and provide a framework for a meeting of Gabor and Choquet when connecting the theory of non-additive measures and integrals with some questions of time-frequency and harmonic analysis. We briefly touch some constructions of non-additive integrals based on super level measures and their usage in different connections.

**Acknowledgement.** This work was supported by the Slovak Research and Development Agency under the contract No. APVV-16-0337

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# On the Oscillation of Second-Order Delay Differential Equations

**Irena Jadlovská**

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The purpose of this talk is to present new sufficient conditions for oscillation of all solutions to the second-order half-linear delay differential equation

$$\left(r(t)\left(y'(t)\right)^{\gamma}\right)' + q(t)y^{\gamma}(\tau(t)) = 0, \quad t \geq t_0,$$

under the condition

$$\int_{t_0}^{\infty} \frac{\mathrm{d}t}{r^{1/\gamma}(t)} < \infty.$$

The results essentially improve, complement and simplify a number of related ones in the literature. Possible extensions for higher-order differential equations with noncanonical operators are briefly discussed.

# Entropy in Physics and the Information Theory

**Zuzana Ontkovičová**

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Entropy is a term first used in physics to describe special property of the thermodynamic system – degree of chaos or degree of disorder. Later, Claude Shannon, who studied communication from the mathematical point of view, based his work on this concept. In 1964 he defined a new quantity – entropy in the information theory according to the interpretation of the physical entropy. In his work, entropy can be described as an uncertainty in a received message when we know what had been sent at the beginning.

Until now, many papers and books analysing this theme have been written. Axiomatic characterization according to the experimentally discovered properties is given in [1], some applications in physics and the information theory are presented in [2] and new types of entropy are introduced in [3, 4].

In the talk, we will present the meaning of the Shannon-Khinchin axioms, general characteristics and connections among various types of entropy and some figurative applications of this quantity in physics and the information theory.

**Acknowledgement.** This work was supported by the Slovak Research and Development Agency under the contract No. APVV-16-0337 and the internal faculty grant No. VVGS-PF-2019-1037.

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# Study of Fuel Parameter Prediction Possibilities

**Beáta Stehlíková**

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Solid fuels are one of the most commonly used sources of heat energy. The fuel components may be considered as mixtures where the sum of the units gives a total or 100 %. The present publication deals with the possibility of predicting the proportion of ingredients. Regression models and linear vector combinations are used. Statistical indicators are used to evaluate the suitability of the methods.

# Cardinal Invariant $\lambda(\mathcal{I}, \mathcal{J})$

**Viera Šottová**

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The cardinal invariant $\lambda(\mathcal{I}, \mathcal{J})$ was introduced by J. Šupina in [2] as combinatorial characteristics of $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)$-space. The selection property $\mathrm{S}_1(\mathcal{I}\text{-}\Gamma, \mathcal{J}\text{-}\Gamma)$ was described by P. Das and it comes as ideal version of selection principles from Scheepers' Diagram.

In the talk we will analyze cardinal invariant $\lambda(\mathcal{I}, \mathcal{J})$, show its role in selection principles and represent it through slaloms. Some of presented results are already published in [1].

**Acknowledgement.** Supported by grants VEGA 1-0097-16 and VVGS-PF-2019-1037.

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[2] Šupina, J.: *Ideal QN-spaces*, J. Math. Anal. Appl. **434** (2016), 477–491.

# Super Level Measure

**Jaroslav Šupina**

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We discuss a concept of super level measure introduced by Y. Do and C. Thiele [2] and studied in its own in [1]. We hope that in addition to original motivation for its introduction it finds its applications in the theory of non-additive integrals.

The aim of the talk is to present several viewpoints on super level measure which may demonstrate the differences of the new concept in comparison with the original one used before.

**Acknowledgement.** Supported by the grant APVV-16-0337 of Slovak Research and Developement Agency APVV.

**References**

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# Compatible Operations on Rings $Z_n$

**Ivana Varga**

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Compatible operations are generalization of polynomials, therefore in algebra they draw a big attention. An $n$-ary operation on an algebra is called compatible if it preserves all congruences on this algebra. Compatible functions form a clone that contains all polynomial operations. If every compatible function on algebra $\mathbf{A}$ is polynomial, then we say that this algebra is affine complete. Especially, for rings $Z_n$ it is known that only for a square-free $n$ this algebra is affine complete.

We would like to investigate compatible functions on rings $Z_n$ that are not affine complete. We would like to describe all clones that are between clones of polynomial and compatible functions. By this research we continue the work started in [1].

**Acknowledgement.** This work was supported by national grant VEGA 1/0097/18.

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# Star Edge Coloring of Grids and Circulant Graphs

**Erika Vojtková**

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A *star edge-coloring* of graph $G$ is a proper edge-coloring without bichromatic paths and cycles of length four. The least number k such that $G$ admits a star edge-coloring with $k$ colors is the *star chromatic index* of $G$. Dvořák, Mohar and Šámal asked if the star chromatic index of complete graphs is linear in the number of vertices and gave a near-linear upper bound. We consider graphs with highly regular structure such as grids and circulant graphs. We improve some existing bounds for the star chromatic index of these classes of graphs and give a number of exact values.

## Program 20. Konferencie košických matematikov

## Programme of the 20th Conference of Košice Mathematicians

**Štvrtok – Thursday 25. 4. 2019**

$12^{00}$ – **Registácia účastníkov – Participants Registration**
$13^{00}$ – **Obed – Lunch**

$14^{00}$ – **Slávnostné otvorenie konferencie – Conference Opening**

$14^{10}$ – Elena Grešová (ÚRIVP FBERG TUKE) *Student's Value of Study Mathematics and Economics*
$14^{35}$ – Zuzana Ontkovičová (ÚMV PF UPJŠ) *Entropy in Physics and the Information Theory*
$15^{00}$ – Erika Fecková Škrabuľáková (ÚRIVP FBERG TUKE) *Mathematical Tool for Suppliers Evaluation*

$15^{25}$ – **Občerstvenie – Coffee-break**

$16^{00}$ – Tadeáš Gavala (ÚMV PF UPJŠ) *Using Card Games in Probability Tasks at Secondary School*
$16^{25}$ – Anton Hovana (ÚMV PF UPJŠ) *Beyond the Hirsch Index*
$16^{50}$ – Viera Šottová (ÚMV PF UPJŠ) *Cardinal Invariant* $\lambda(\mathcal{I}, \mathcal{J})$
$17^{15}$ – Ivana Varga (ÚMV PF UPJŠ) *Compatible Operations on Rings* $Z_n$

$18^{00}$ – **Večera – Dinner**

**Piatok – Friday 26. 4. 2019**

$7^{30}$ – **Raňajky – Breakfast**

$8^{30}$ – Otvorenie 2. dňa – Opening Day 2 — predstavenie 1. pozvaného prednášajúceho – Introducing the 1st Invited Lecturer

$8^{35}$ – František Kuřina (UHK Hradec Králové) *Mathematics as a Part of Culture*

$9^{30}$ – Predstavenie 2. pozvaného prednášajúceho – Introducing the 2nd Invited Lecturer

$9^{35}$ – Josef Molnár (KAG PrF PU Olomouc) *Mathematical School Terminology Yesterday, Today and Tomorrow*

$10^{30}$ – **Občerstvenie – Coffee-break**

$11^{05}$ – Predstavenie 3. pozvaného prednášajúceho – Introducing the 3rd Invited Lecturer

$11^{10}$ – Jan Andres (KMAaAM PrF PU Olomouc) *Sharkovsky Theorem and Differential Equations*

$12^{05}$ – Irena Jadlovská (KMTI FEI TUKE) *On the Oscillation of Second-Order Delay Differential Equations*

$12^{30}$ – **Obed – Lunch**

$13^{45}$ – Prekvapenie konferencie, konferenčné fotenie – Conference Surprise, Conference Photo — Predstavenie 4. pozvaného prednášajúceho – – Introducing the 4th Invited Lecturer

$14^{00}$ – Lev Bukovský (ÚMV PF UPJŠ) *Finity and Infinity (in Mathematics)*

$14^{55}$ – **Občerstvenie – Coffee-break**

$15^{30}$ – Predstavenie 5. pozvaného prednášajúceho – Introducing the 5th Invited Lecturer

$15^{35}$ – Martin Bača (KAMI SjF TUKE) *Local Antimagic Vertex Coloring of Graphs*

$16^{30}$ – Predstavenie 6. pozvaného prednášajúceho – Introducing the 6th Invited Lecturer

$16^{35}$ – Jakub Przybyło (AGH UScT Kraków) *Adjacent Vertex Distinguishing Edge Colourings*

$17^{30}$ – Beata Stehlíková (ÚRIVP FBERG TUKE) *Study of Fuel Parameter Prediction Possibilities*

$18^{00}$ – **Večera a spoločenský večer – Dinner & Party**

**Sobota – Saturday 27. 4. 2019**

$7^{30}$ – **Raňajky – Breakfast**

$8^{20}$ – Otvorenie 3. dňa – Opening Day 3

$8^{25}$ – Erika Vojtková (ÚMV PF UPJŠ) *Star Edge Coloring of Grids and Circulant Graphs*

$8^{50}$ – Jozef Doboš (ÚMV PF UPJŠ) *Using GeoGebra to Solving Equations and Inequalities*

$9^{15}$ – Katarína Cechlárová (ÚMV PF UPJŠ) *Modelling the Transplant Waiting List*

$9^{40}$ – **Občerstvenie – Coffee-break**

$10^{05}$ – Ondrej Hutník (ÚMV PF UPJŠ) *Meeting Gabor and Choquet Somewhere in an Outer Measure Space*

$10^{30}$ – Jaroslav Šupina (ÚMV PF UPJŠ) *Super Level Measure*

$10^{55}$ – **Záver konferencie – Conference Closing**

$11^{00}$ – **Obed – Lunch**

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Zostavovatelia: Ján Buša, Erika Fecková Škrabuľáková
Vydavateľ: Technická univerzita v Košiciach
Rok: 2019
Vydanie: prvé
Náklad: 55 ks
Rozsah: 32 strán
Vydané v Košiciach, 2019
Elektronická sadzba programom pdfLaTeX
Tlač: Univerzitná knižnica TUKE, Němcovej 7, 042 00 Košice

**ISBN 978-80-553-3293-2**