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Predhovor

Vážení priatelia, milí hostia, kolegyne a kolegovia,

vitajte na 15. Konferencii košických matematikov. Túto konferenciu organizuje Jednota slovenských matematikov a fyzikov, pobočka Košice, v spolupráci s Ústavom matematických vied Prírodovedeckej fakulty UPJŠ, katedrami matematiky Technickej univerzity a pobočkou Slovenskej spoločnosti aplikovanej kybernetiky a informatiky pri KRVP FBERG TU Košice. Konferencia sa koná, tak ako aj jej predchádzajúce ročníky, v útulnom prostredí Učebno-výcvikového zariadenia TU Košice – v Herľanoch.

15. konferencia je dôvodom na také malé jubileum. Myšlienka organizovať konferenciu tohto typu nás napadla pred viac ako pätnástimi rokmi. Cítili sme, že ľudia živiaci sa matematikou v jej rôznych podobách (učitelia, vedci, aplikovaní matematici) a žijúci na východe Slovenska by mali mať možnosť sa pravidelnejšie stretávať, podeliť sa s rovnako „postihnutými“ kolegami o svoje radosti i starosti súvisiace s prácou matematika či matematikára; následne spoločne alebo s ďalšími spriaznenými dušami hľadať riešenia či východiská z problémov. Prípadne si vzájomne pomáhať a povzbudiť sa navzájom. Ďalej sme mali predstavu, že by to mala byť seriózna konferencia s kvalitným obsahom, najmä pozvanými prednáškami. Od začiatku sme na ňu pozývali prednášajúcich s cieľom, aby to boli či už zrelé alebo práve vychádzajúce kvalitné osobnosti, známe vo svojom prostredí, s cieľom dozvedieť sa nové veci, nadviazať nové či upevniť staré kontakty. Viaceré z týchto prednášok mali taký pozitívny ohlas, že ich autori boli pozvaní prednášať aj na iných konferenciách.

To, že Konferencia košických matematikov sa koná po 15. krát je len potvrdením, že naše myšlienky našli úrodnú pôdu. Každoročne sme na nej mali skvelých prednášajúcich. Na výbere a príprave konferencie sa pracuje celý rok. O výbere pozvaných prednášajúcich sa v podstate rozhoduje na tradičnom každoročnom stretnutí predsedov košickej pobočky JSMF s košickými profesormi matematiky a vedúcimi košických matematických pracovísk, vrátane riaditeľa Gymnázia na Poštovej ulici, ktoré má matematické triedy.

Za tých 15 rokov sa vykryštalizovala aj štruktúra konferencie. Prvé dva dni (streda a štvrtok) sú venované mladým začínajúcim matematikom. Mnohí dnes už veľmi úspešní kolegova mali svoje prvé verejné odborné či vedecké vystúpenie práve na našej konferencii. Vystúpenia mladých kolegov majú z roka na rok vyššiu úroveň, čo organizátorov veľmi teší. V piatok a v sobotu dopoludnia sa konajú najmä pozvané prednášky, aby sa na nich mohlo zúčastniť čo najviac účastníkov. Spoločenský piatkový večer je organizovaný tak, aby bolo možné v menších skupinách pri pohári vínka predebatovať rôzne otázky.

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Obsah – Contents

17^{00} – Erika Škrabuľáková (ÚRaIVP FBERG TUKE) *On the Total Thue Colourings of Graphs*
17^{20} – Sergej Ševec *Some Remarks on Block Inscribability of Polyhedra*

18^{00} – **Večera – Dinner**

19^{00} – **Workshop:** Patrik Zakuťanský (FBERG TUKE) *LEPC – a Better Registration of Publications and Citations*

Piatok – Friday 4. 4. 2014

8^{20} – Stanislav Jendroľ (ÚMV PF UPJŠ) *Cyclic Words and Edge Colourings of Plane Graphs (or How to Discover New Theorems)*
9^{10} – Ján Mazák (PdF TU Trnava) *Perfect Matchings in Cubic Graphs*

10^{00} – **Občerstvenie – Coffee-break**

10^{30} – Bohdan Datsko (IAPMM, NASU, Lviv, Ukraine) *Mathematical Modeling of Self-Organization Phenomena in Nonlinear Dynamical Systems with Classical and Fractional Derivatives*
11^{20} – Jozef Fecenko (KMHI EUBA) *On Actuarial Profession and Actuarial Mathematics*
12^{10} – Marián Ružička (Gymnázium Poštová Košice) *40^{th} Anniversary of Classes Focused on Mathematics at Gymnasium Poštová Str. 9, Košice*

12^{30} – **Obed – Lunch**

14^{00} – Marie Demlová (KM FEL ČVUT Praha) *Discrete Mathematics at FEE, CTU in Prague*
14^{50} – Edita Pelantová (KM FJFI ČVUT Praha) *Exotic Representations of Numbers*

15^{40} – **Občerstvenie – Coffee-break**

16^{10} – Miroslav Haviar (KM FPV UMB, Banská Bystrica) *Prime Numbers and Their Numeric Relatives*
17^{00} – Ladislav Spišiak (Gymnázium Šrobárova Košice) *Mathematical Didactics in Old Way? (Objectives of Education and Key Competences)*

18^{30} – **Večera a spoločenský večer – Dinner & Party**

Invited lectures

Mathematical Modeling of Self-Organization Phenomena in Nonlinear Dynamical Systems with Classical and Fractional Derivatives

Bohdan Datsko

Institute of Applied Problems of Mechanics and Mathematics, National Academy of Sciences of Ukraine, Lviv, Ukraine; *and* Technical University of Kosice, Slovakia

The mathematical models and methods that make it possible to investigate the nonlinear differential equations with integer and fractional derivatives and theoretically explain and explore the self-organisation phenomena in nonequilibrium media are presented.

For nonlinear systems with classical derivatives the main attention is paid to applied mathematical models. Based on reaction-diffusion mathematical models the formation of stationary and spatio-temporal dissipative structures in physical, chemical and biological systems are demonstrated. For nonlinear systems with derivatives of rational order the main types of mathematical models and their applications for media with anomalous diffusion and heriditary are considered.

The theoretical analysis of possible instabilities in fractional nonlinear systems is considered. The new types of nonlinear solutions for the basic reaction-diffusion systems with fractional derivatives and new types of limit cycles for simplest fractional systems of ordinary differential equations are obtained.

Acknowledgement. This work is supported by the National Scholarship Program of the Slovak Republic.

Program 15. Konferencie košických matematikov

Programme of the 15th Conference of Košice Mathematicians

Streda – Wednesday 2. 4. 2014

12^{30} – **Obed – Lunch**

14^{35} – **Otvorenie konferencie – Conference opening**

14^{40} – Zuzana Farkasová (ÚMV PF UPJŠ) *About Lattices of Convex Subsets of Monounary Algebras*

15^{00} – Miroslava Šuličová (ÚMV PF UPJŠ) *Monounary Algebras and Their Centralizers*

15^{20} – Michal Dečo (ÚMV PF UPJŠ) *Strongly Dominating Sets and Laver's Trees*

15^{40} – Anna Mišková (ÚMV PF UPJŠ) *Localization Operators Associated with the Stockwell Transform*

16^{00} – **Občerstvenie – Coffee-break**

16^{30} – Lenka Halčinová (ÚMV PF UPJŠ) *On τ-Decomposable Measures and Integrals*

16^{50} – Jana Molnárová (ÚMV PF UPJŠ) *On the Seminormed Integral*

17^{10} – Martina Juhásová (ÚMV PF UPJŠ) *Statistics in School Mathematics*

17^{30} – Noémi Székelyová (ÚMV PF UPJŠ) *Early Algebra in the Primary School*

18^{00} – **Večera – Dinner**

On Actuarial Profession and Actuarial Mathematics

Jozef Fecenko

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Faculty of Economic Informatics, University of Economics,
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Actuaries are experts that apply mathematical and statistical methods to assess risk in insurance, finance and other industries and professions. The current trend in actuarial science is heading towards significant use of stochastic methods in comparison with deterministic ones. Application of stochastic models in actuarial sciences and subsequently in the insurance practice is explicitly recommended by the EU Solvency II project. In the first part of contribution, we discuss questions such as definition of actuary, what does an actuary do, what does tells us the Slovak Insurance Act on responsible actuary, what are the main industry sectors where actuary could work in and what skills does actuary need. At the end of this part is shown that actuary was proclaimed to be the best job in USA according methodology Jobs Rated. We also mentioned on Slovak Society of Actuaries and international actuarial associations.

The Actuarial Mathematics is very broad topic. For a short time we had to select only some fundamental fields of Actuarial Mathematics: fundamental principles of non-life insurance and their mathematical representations, the individual risk model, the collective risk model and ruin probabilities, life insurance, life tables, modelling of mortality, modern methods of valuation in live insurance, pension and health insurance.

Acknowledgement. The contribution was supported by the grant VEGA, No. 1/0806/14, "SCR calculation to cover the risks of non-life insurance in accordance with practical needs".

Acknowledgement. Partially supported by the grant 1/0002/12 of Slovenská grantová agentúra VEGA.

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Workshop: LEPC – a Better System for Registration of Publications and Citations

Patrik Zakuťanský, Igor Podlubný

BERG Faculty, Technical University of Košice

At this workshop we will introduce the system called LEPC (abbreviation of "Lepšia Evidencia Publikácií a Citácií"), which has been developed at the BERG Faculty of the Technical University of Kosice as a (better) alternative to the existing all-Slovakian system EPČ. In contrast to the existing EPČ system, the presented LEPC system provides features, which users – teachers and researchers at the Slovak universities – really need, including the following:

- import of data (lists publications, lists of citations) from the standard databases – Web of Science, Scopus, and numerous other publication and citation databases and sources;
- export of data (lists publications, lists of citations) to the really usable BibTeX format;
- efficient multi-level prevention of creation of duplicate records about the same publication;

Primes and Their Number Relatives

Miroslav Haviar

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We mention Euklid's proof of the existence of infinite number of primes via the concept of a primorial number and an open problem of the existence of infinite number of primorial primes. To support that primes "occur often" we present Dirichlet's theorem, Prime twin conjecture and Goldbach's conjecture and to support that primes "occur rarely" we argue with arbitrarily big prime gaps and mention jumping champions. The opinion that we still know little about primes is supported by about a dozen of open problems presented during the lecture. Among the problems already solved we mention the Erdös problem solved by Green and Tao, which concerns the existence of n primes in an arithmetical sequence for any natural n. The opinion that we know relatively enough about primes is supported by Gandi's (recursive) formula for the n-th prime and we explain why it works. We mention pseudoprimes and AKS algorithm and also Fermat's numbers and Eisenstein's conjecture regarding the existence of infinitely many Fermat's primes. We mention Euler's and Goldbach's arguments for the infinite number of primes while showing how the latter uses Fermat's numbers. Towards the end we mention our own investigation with P. Mališký (`http://actamath.savbb.sk/pdf/acta1502.pdf`) of so-called superprimes and several related conjectures. We finally mention our conjecture concerning what we call a Generalised Dirichlet's theorem, from which two well-known conjectures about the existence of infinite number of Fermat's primes and Mersenne's primes follow rather easily as consequences.

Web Preferences Using Robust Fuzzy Discrete Dynamic Systems

Lukáš Štefanský and Ján Plavka

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The main topic of this article is a description of a proposal of web shop page based on user preferences searching. Learning user preferences is a complex area, especially difficult for performing experiments – every person is different and has different preferences, which often change in time. The problems studied in this work are based on the idea of helping user in making decisions. The main motivation lies in e-commerce, where user might benefit from a recommendation of objects that might interest him without needing to process all objects manually. We start with author diploma thesis, where is presented basic web preference portal, based on knowledge from max-min algebra. It uses attributes like robustness and interval preference matrixes for modelling users needs and notions. We present an various algorithms for checking robustness (possible or universal) of specific matrixes like circulant, Hankel-circulant and even for interval matrixes. Finally we propose extensions for future to make this model more suitable for real world usage and more adaptable for different kinds off objects (goods, services, insurances etc.).

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Consider a 2-connected plane graphs. All its faces are bounded by cycles, called the *facial cycles*. If we label all the edges of a 2-connected plane graph G with letters from an alphabet $\mathbb{A}$, then any face $\alpha = [e_1, e_2, \ldots, e_k]$ determined by the edges $e_1, e_2, \ldots, e_k$ can be associated with a cyclic word $a_1a_2 \ldots a_k$, where k is size (degree) of the face α and a_i is a labell of the vertex v_i. The word $a_1a_2 \ldots a_k$ is called the facial word of the face α of G.

In our talk we will consider the following problem:

Problem: What is the minimum number of letters in an alphabet $\mathbb{A}$ that allows to label the edges of a given 2-connected plane graph G in such a way that all the facial words of G over $\mathbb{A}$ have a given property $\mathcal{P}$?

We will give a survey on results and open questions concerning this problem for several properties of words.

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Perfect Matchings in Cubic Graphs

Ján Mazák

PdF Trnavská univerzita, Trnava, Slovakia

Problems related to perfect matchings have been an integral part of graph theory since its very beginnings. Petersen proved in 1891 that every bridgeless cubic graph can be decomposed into a 1-factor and a 2-factor. Such decompositions provide crucial information about the structure of the graph and are used in a wide variety of applications ranging from colourings to approximation algorithms for NP-hard problems.

Albeit their existence is guaranteed, it is notoriously hard to find perfect matchings in cubic graphs — only very recently it was proved that the number of perfect matchings is exponential. And if you need a perfect matching with a special property, you are touching the deepest and most intriguing problems in graph theory.

for all n where such graphs exist. On the other hand, for $k = 4$, there exist infinitely many graphs for which this ratio is 1. We study the estimates of possible limit value $\lim_{n\to\infty} r_u(n, \mathcal{G})$ for the classes of 5-, 6- and 7-critical graphs; in addition, we show that, for each rational number $\rho \in \langle \frac{5}{3}, \frac{9}{5} \rangle$, there exists a 4-critical planar unit-distance graph with density ρ, where the embedding is faithful; moreover the bound $\frac{5}{3}$ is best possible.

Some Remarks on Block Inscribability of Polyhedra

Sergej Ševec

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The notion of block inscribability of polyhedra is introduced. The relevancy of this approach is disscussed thoroughly. Consequently, some interesting classes of polyhedra and problems are focussed. In particular, the apparatus devised by Dillencourt for affirming or assuring inscribability is developed further to encompass gradual difficulties as are involved in the presented selection of basic polyhedral blocks. To define representative examples and situations, also the other apparatus is used, incepted originally in the theorem on noninscribability of polyhedra by Steinitz, as well in its original as properly generalized forms.

class and their teacher of mathematics and a class teacher as well RNDr. Katarína Grünmannová were at the inception of this tradition in the academic year 1974/75 and started the history of the mathematical gymnasium (in Slovak gymnázium) – in that time one of the two in Slovakia.

The aim of establishing these special classes was to support students gifted at mathematics and provide them with the possibility of intensive development of their talent, focus them on the study at universities where mathematics is a dominant subject and to improve the results of the Slovak national team at International Mathematics Olympiad. The first teachers of mathematics at these classes were RNDr. Katarína Grünmannová and Michal Ščerbák. The guarantor of the quality assigned by the Ministry of Education was professor RNDr. Ernest Jucovič from the Natural Science Faculty of University of Pavol Jozef Šafárik in Košice. The first external head of the Mathematics Olympiad Club was professor RNDr. Lev Bukovský, DrSc. and he was followed by other teachers of the faculty. Within the four decades there were generations of school-leavers and we are pleased to say that many of them decided to follow their teachers in their profession and are university teachers at the Natural Science Faculty of University of Pavol Jozef Šafárik in Košice nowadays or at Mathematical Instituite of Slovak Academy of Sciences, namely RNDr. Jozef Jirásek, PhD; doc. RNDr. Ivan Žežula, CSc; doc. RNDr. Roman Soták, PhD; doc. RNDr. Stanislav Krajči, PhD.; RNDr. Vladimír Dančík PhD., doc. RNDr. Tomáš Madaras PhD., RNDr. Igor Fabrici Dr. rer. nat., and others.

Mathematical Didactics in Old Way? (Objectives of Education and Key Competences)

Ladislav Spišiak

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Although the notion of key competences plays the central role in the contemporary so called transformation of education, the aim of this lecture is to turn the attention onto another important didactic notion – educational objectives – as a useful and even necessary tool in education. The well-known hierarchy of educational objectives (performance, skills, knowledge, attitudes) enables the teacher to distribute purpously his activity into all these domains which is a necessary condition for the achievement of proper

A Game, which Incidentally Teaches Algebra

Izabela Solarz

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Children have many fundamental problems with learning algebra. It is also difficult for teachers to carry their students through the threshold of algebraic symbols and operations. On the other hand most of our students love computer games and they can spend a lot of time on playing. Is it possible to overcome difficulties in learning Math by computer applications? With a teacher keeping an eye on his students? DragonBox is an example of such a game. It unnoticeably takes children into algebraic world of symbols. They use colour pictures to put them into different places according to the clear rules. Then very slowly, the pictures are replaced by numbers and letters. The game encourages children to be active, forces to pay attention, to come back and look for the best solution. After this amusing entrance to the basic equations, students can start solving them on a piece of paper, but then the symbols: letters, numbers, operation signs don't frighten them. Algebraic rules are now familiar game rules. As a Math teacher I decided to try out DragonBox as a tool for helping students with difficulties in mastering the solving of simple linear equations. In my contribution I'll show videos recorded in the classroom and talk about my observations and my impression. I 'll also put important questions concerning this way of learning algebra as a game with rules consisting on bare manipulation of symbols.

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provoke students to in-depth reflection on the reasoning in mathematics. In particular, we consider the role of empirical reasoning, intuitive and formal reasoning in mathematics; ability to conduct various types of evidence reasoning (deductive, reductive, indirect proof), etc. These components of methodological knowledge are necessary for mathematics teacher to verify the correctness of pupils' reasoning and develop their skills of mathematical arguing and proving. Importance of these skills was pointed out by some creative mathematicians (eg. Steinhaus, 1969; Thurston, 1994).

The first stage of implementation of our conception is diagnosis of students' skill in evaluating the correctness of a mathematical reasoning. This step is developing in cooperation with representatives of Oddelenie Didaktiky Matematiky PF UPJŠ in Košice. During the conference we will present preliminary results of research.

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minimum cardinality and maximum cardinality POM. We show that the former problem is NP-complete even in the one-to-one markets with the preference list of each applicant containing at most two entries. For the latter problem we show that, although it is polynomially solvable in the special one-to-one case, it is NP-complete for many-to-many markets.

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About the Probability in the School Classroom

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Pupils' misconception of probability is often caused due to lack of their experiences. We introduce a real problem and a possible method for solving this task without the necessary mathematical background. For this purpose we reformulate the given problem. Consequently we use a simulation for finding an approximate solution.

The aim of this proposal is to provide a way how to give students or pupils valuable experience with experimentation, recording and evaluation of some type of data. In view of the usual methodology for teaching probability, we also analyze several tools used for the simulation and the solution of the proposed problem.

Acknowledgement. This work was supported by Slovak Research and Development Agency under the contract No. APVV-0482-11, by VEGA grants No. 1/0497/11 and No. 1/0729/12, and KEGA grant No. 040TUKE-4/2014.

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Unique Maximum Colourings of Plane Graphs

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A unique-maximum k-edge-colouring with respect to faces of a 2-edge-connected plane graph G is an edge-colouring with colours from the set $\{1, 2 \dots k\}$ such that for each face f of G the maximum colour occurs exactly once on the edges of f. The minimum number k that there exists a unique-maximum k-edge-colouring of graph G is called unique maximum index. We will prove that any 2-edge-connected plane graph has such a colouring with 3 colours in general and with 6 colours if we require the colouring to be facially-proper. We will also discuss relations of these colourings to other types of colourings of plane graphs.

Keywords: plane graph, unique-maximum colouring, edge colouring

About Lattices of Convex Subsets of Monounary Algebras

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We deal with lattices of all convex subsets of monounary algebras. A subset B of A is called convex (in (A, f)) if, whenever $a, b_1, b_2 \in A$ are distinct elements such that $b_1, b_2 \in B$ and there is an oriented path (in the graph $\mathcal{G}(A, f)$) going from b_1 to b_2, not containing the element b_2 twice and containing the element a, then a belongs to B as well. The system $\mathrm{Co}(A, f)$

Mathematical Solution of a Moisture Drying System for Porous Building Materials

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Presence of moisture in the building materials leads to a larger energy loss, especially in winter; therefore, drying is important to improve the energy efficiency of buildings. The drying process is based on the evaporation of the liquid water on the material surface where the water vapor releases to surrounding, or it is based on the evaporation in the pores of the material and subsequently moving through the pores to the surface and surrounding. We derive a mathematical model that describes liquid water and vapor diffusion in a wet material as two separate processes. We also present an exact solution of this model and compare it with the classical moisture transfer solution representing transfer of both liquid water and vapor as a single moisture variable. Finally, we present the calculation of diffusion coefficient and compile the values for various building materials. The model allows considering the drying in various materials as two independent processes for transfer of liquid water and vapor. Energy losses can be calculated using the model depending on the moisture content in the materials. The system is solved by means of the variables separation method.

This problem end methodology were published in the recent work [1]. Other aspects of the moisture drying problem were studied in [2], and [3].

Acknowledgement. This work was supported by grant APVV SK-AT-0008-10.

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On Maximum Weight of a Planar Graph of Given Density

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The weight of an edge of a graph is defined to be the sum of degrees of vertices incident with this edge. The weight of a non-empty graph G, $w(G)$, is the minimum of weights of all edges of G.

We will show that if G is a planar graph with density at least two, then $w(G) < \frac{9\rho(G)-12}{\rho(G)-2}$. Moreover, we will construct planar graph G with weight $\left\lfloor \frac{9\rho(G)-12}{\rho(G)-2} \right\rfloor - 1$.

Optimal Drawings of $K_{5,n}$

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Zarankiewicz's Conjecture (ZC) states that the crossing number $\mathrm{cr}(K_{m,n})$ equals $Z(m,n) := \lfloor \frac{m-1}{2} \rfloor \lfloor \frac{m}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor \lfloor \frac{n}{2} \rfloor$. Since Kleitman's verification of ZC for $K_{5,n}$ (from which ZC for $K_{6,n}$ easily follows), very little progress has been made around ZC and the most notable exceptions involve computer-aided results. With the aim of gaining a more profound understanding of this notoriously difficult conjecture, [1] classifies the optimal (that is, crossing-minimal) drawings of $K_{5,n}$.

In this talk we introduce the definition of vertex rotation $\mathrm{rot}_D(a)$ for a 5-degree vertex a in a drawing D of $K_{5,n}$, that is a cyclic permutation of the labels corresponding to the 5 vertices adjacent to a in D. We also recall from [1] the definition of $\mathrm{Rot}_M D$ that is the multiset containing the

analogue of Kotzig theorem cannot be extended in general for graphs of minimum degree 2. The graph $K_{2,r}, r \geq 2$, has girth exactly 4. The *girth* of a graph is the length of a shortest cycle in the graph. We prove that every 2-connected plane graph of girth at least five has an edge with degree sum of its endvertices at most 7. This bound 7 is the best possible if girth is exactly 5 or 6. For higher girth, the constant 7 can be lowered, e.g., to 4 if girth is at least 11.

Keywords: planar graph, girth of graph, weight of edge, Kotzig's theorem

Localization Operators Associated with the Stockwell Transform

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In our contribution we deal with an important transform, the so-called continuous Stockwell transform, thus providing the last piece of the mosaic of a picture of the time-frequency methods in time-frequency analysis: the continuous wavelet transform, the short-time Fourier transform and the continuous shearlet transform. Recently, these three mentioned transforms were examined in terms of decomposition of the corresponding function spaces and from the behavior of associated Toeplitz localization operators. Therefore, we introduce the operator S_φ of continuous Stockwell transform (with respect to an admissible window function φ) acting on a signal from $L_2(\mathbb{R})$, and present its relationship with other time-frequency transforms. Using the Vasilevski scheme we describe the structure of the image space of $L_2(\mathbb{R})$-functions under the transform S_φ, which allows us to study in detail the Toeplitz localization operators based on S_φ. We construct a Bargmann-type transform mapping the Stockwell-Toeplitz operator T_a^φ with a symbol a depending on the second coordinate in the plane to a multiplication operator with an explicitly computable symbol, and to a certain pseudo-differential operator in the case of general symbols. This allows us to study properties of the corresponding localization operators and of the algebras generated by them.

where τ is a triangle function on Δ^+ and ε_0 is the distribution function of Dirac random variable concentrated at point 0.

The aim of this contribution is to present a Lebesgue-type approach to the integration of non-negative real-valued functions with respect to the probabilistic τ-decomposable measures. We investigate basic properties of the corresponding probabilistic integral and discuss how the properties of the underlying triangle function affect these properties. The above results are included in the recent paper [3].

Acknowledgement. The present work was supported by the Internal grant VVGS-2013-121, VVGS-PF-2013-115.

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Monounary Algebras and Their Centralizers

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For a given (partial) algebra $\mathcal{A}$, its centralizer is defined as the set of those transformations of $\mathcal{A}$ into $\mathcal{A}$, which commute with all fundamental operations of $\mathcal{A}$. Further, a second centralizer is the set of all transformations which commute with all elements of the (first) centralizer. We characterize the monounary algebras having the property that the first and the second centralizer coincide.

Statistics in School Mathematics

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In the age of information, the ability to manage and make sense of the large amount of data encountered in everyday life is important skill to acquire. Graphs and statistics bombard the public in areas such as advertising, opinion polls, population trends, health risk, and progress of students in schools. Statistics involves using data in the form of numbers and graphs to describe our world. Statistical literacy is critical to understanding the world around us, essential for effective citizenship, and vital for developing the question information presented in the media. Misuse of statistics occurs even in trustworthy sources like newspapers, where graphs are often designed to exaggerate a finding.

In recent years, there have been an expansion and renewal of the statistics content in the mathematics curricula in many countries through all school grade levels from primary to secondary levels. Children start studying basic statistical concepts at elementary school and continue to develop these concepts in curriculum until secondary school, where students may study elements of statistical inference. Students in school should have meaningful experiences with basic concepts of statistics throughout their school years. We present some activities for introducing of selected measures of center which can help students to understand them. We also focuse on students´ ability to extract qualitative information from quantitative information and to create new information from qualitative and quantitative information.

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The Issue of Irrational Numbers Teaching at High Schools

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The topic of real numbers and especially of irrational numbers is quite unpopular for grammar school students and also for teachers of mathematics. There are many reasons for that. Lack of time devoted to mathematics reserved for study of real numbers, unattractive tasks of a proof nature, low learning materials.

In recent years, the topic of real numbers also progressively slides out of the first classes of grammar schools up to college, where it is assumed that the student is familiar with this area of mathematics.

We show how to make appropriate use of mathematical software for the visualization of the difference between rational and irrational numbers in this paper. We introduce a proposal visualization of the Pythagorean Theorem on the measurable base and we also show how to demonstrate the difference between rational and irrational numbers on the example with a barcode.

Light Edges in Plane Graphs of Given Girth

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Let G be a planar graph. The well-known Kotzig theorem states that every 3-connected planar graph contains an edge such that the sum of the degrees of its endvertices is at most 13. This result was extended by D. Barnette and independently by O. Borodin to all planar graphs having minimum degree 3. The plane graph $K_{2,r}, r \geq 2$, has served as an example that

m cyclic permutations associated to each 5-degree vertex in D (in the same chromatic class). This in turn provides the necessary elements to stablish an isomorphism which reads that two drawings D and D' of $K_{5,n}$ are isomorphic if there exists a relabelling of the 5 n-degree vertices in D such that $\mathrm{Rot}_M D \equiv \mathrm{Rot}_M D'$. We also exhibit the two-parameter family of optimal drawings $D_{r,s}$ of $K_{5,4(r+s)}$ (for $r, s \geq 0$), with no antipodal vertices found in [1] which are not isomorphic to the widely known natural drawings of $K_{m,n}$ (the so-called *Zarankiewicz drawings*). Then we set up the classification of all the drawings of $K_{5,n}$ for n even that is, if n is even, then every optimal drawing of $K_{5,n}$ is (i) a Zarankiewicz drawing or (ii) an antipodal free drawing isomorphic to some $D_{r,s}$ and $4(r+s) = n$ or (iii) a superimposition of Zarankiewicz drawings with a drawing isomorphic to $D_{r,s}$ for some non-negative integers r, s

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On τ-Decomposable Measures and Integrals

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Triangle functions are appropriate candidates for addition in the so-called probabilistic metric spaces. Also, they play a crucial role in generalization of previously defined concepts of τ_T-measures [1], and $\tau_{L,A}$-measures [2]. More precisely, the τ-decomposable measure is a set function γ defined on a ring Σ of subsets of a non-empty set Ω taking values in the set Δ^+ of all distribution functions of non-negative random variables satisfying two conditions

(i) $\gamma_\emptyset = \varepsilon_0$;

(ii) $\gamma_{E\cup F} = \tau(\gamma_E, \gamma_F), \quad E, F \in \Sigma,\ E \cap F = \emptyset,$

On the Seminormed Integral

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E. P. Klement, R. Mesiar and E. Pap introduced in [1] a new concept of the so-called universal integral which can be defined for arbitrary measurable spaces, arbitrary monotone measures and arbitrary measurable functions. In our talk we are interested in a class of universal integrals known as *seminormed integrals* which includes the well-known Sugeno as well as Shilkret integral. Indeed, the seminormed integral is the smallest $[0,1]$-valued universal integral having S as the underlying semicopula and is given by

$$\mathbf{I}_S(m, f) := \sup_{t\in[0,1]} S(t, h_{m,f}(t)),$$

where $m : \mathcal{A} \to [0,1]$ is a capacity on a σ-algebra of subsets of a non-empty set X, and $f : X \to [0,1]$ is an $\mathcal{A}$-measurable function. Recall that a binary semicopula $S : [0,1]^2 \to [0,1]$ is a non-decreasing function in both coordinates with neutral element 1 satisfying the inequality $S(x,y) \leq \min\{x,y\}$ for each $x, y \in [0,1]$. A single function $h_{m,f} : [0,1] \to [0,1]$ given by $h_{m,f}(t) := m(\{x \in X;\ f(x) \geq t\})$ provides an "aggregation" of information contained in a pair (m, f). We present some equivalent formulations for the integral $\mathbf{I}_S$, its characterizations and basic properties as well as convergence theorems for $\mathbf{I}_S$-integral sequences. Also, we present several open problems.

Acknowledgement. The present work was supported by grants VVGS-PF-2013-115 and VVGS-2013-121.

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of all convex subsets of a monounary algebra (A, f) is ordered by inclusion and it is a lattice.

We study monounary algebras (A, f), for which the lattice $\mathrm{Co}(A, f)$ is distributive, modular, semimodular, selfdual and complemented, respectively. Further, we describe necessary and sufficient conditions for a lattice L under which there exists a monounary algebra (A, f) such that L is isomorphic to $\mathrm{Co}(A, f)$.

The Use of Streaming Video to Support Teaching and Learning Math Word Problems

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Practice and experience of teachers show that students have problems to solve math word problems. Usually, there are various stages of solving word problems, in which students fail. Some students stop to solve a word problem at very beginning. Even if they know the strategies, some of them are not able to find a right way to solve the specific situation.

Mistakes are natural effects which accompany the learning process. Good mathematics teachers do not regard as a knowledge handicap, but as a rich information source to better understand the student's level of thinking. Moreover a student can benefit from mistakes (he/she can learn on someone else's mistakes) and utilize them for the development of his/her abilities (competencies on level of reflection). In order for pre-service teachers to increase their work efficacy with respect to dealing with student's mistakes, it is necessary for them to analyze the solutions of students, which could contain mistakes of different quality levels.

We introduce interactive videos of student's solutions (right or wrong) of word problems recorded with SmartPen technology which will be a helpful tool for students, teachers and pre-service teachers.

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On Supermagic Joins of Graphs

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A graph is called supermagic if it admits a labeling of the edges by pairwise different consecutive integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. We will deal with supermagic joins of two regular graphs.

Pareto Optimality in Many-to-Many Assignment Problems

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Consider a many-to-many matching market that involves two finite disjoint sets, a set of applicants A and a set of courses C. Each applicant has preferences on the different sets of courses she can attend, while each course has a quota of applicants that it can admit.

In this paper, we examine Pareto optimal matchings (briefly POM) in the context of such markets. Our model is more general and involves many additional constraints e.g., each course bearing some cost and each applicant having an available budget, time restrictions of applicants or mutually exclusive courses. We provide necessary and sufficient conditions for a many-to-many matching to be Pareto optimal and show that checking whether a given matching is Pareto optimal can be accomplished in $O(|A|^2 \cdot |C|^2)$ time. Moreover, we provide a generalized version of serial dictatorship (briefly GSD) and our model can be used to obtain an order for GSD of any many-to-many POM.

For economic reasons (e.g. minimizing the fixed costs of open courses or maximizing the number of assigned applicants), it is interesting to study

Decay Centrality

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The centrality index is a real-valued function which helps us to quantify the role that a given object plays in the network. Decay centrality is a centrality measure based on the proximity between a choosen vertex and every other vertex weighted by the decay. More precisely, decay centrality of a given vertex x of a graph G is define as the sum $\sum_{y \in V(G)} \delta^{d(x,y)}$ where $d(x, y)$ denotes the distance between x and y and $\delta \in (0, 1)$ is a parameter.

We study the general properties of decay centrality, the stability of vertex ranking depending on the choise of parameter δ and we look for the graphs whose vertices do not change their mutual position according to this measure.

Strongly Dominating Sets and Laver Trees

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The notion of strongly dominating subset of the Baire space was introduced in the paper [2] and studied in papers [3] and [1]. In particular in [1] it was shown that the σ-ideal of sets which are not strongly dominating and Laver ideal l^0 are not equal. We present a few observations in an attempt to find out under what conditions this result remains true in more general setting.

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Engaging Middle School Students in Mathematics

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In many students' opinion mathematics is boring, hard to understand and has little to do with real life. One of the reasons for such beliefs is the fact that what we teach in school often has little to do with real mathematics. In last decades educators have indicated a need to make school mathematics closer to what professional mathematicians regard as their subject. The focus should be on thinking, exploring, asking questions, making conjectures and looking for patterns, not just on memorizing facts and formulas and doing exercises. There are many ways to incorporate such activities into middle school math lessons. Among other advantages, such an approach greatly enhances active learning. Unfortunately, as current researches in different countries show, in many classrooms pupils are taught to apply standard methods most of the time.

This talk will describe some ways that middle school teacher can help students experience real mathematics, especially by establishing classroom norms and choosing appropriate problems. Selected tasks from literature and examples of students' work will be presented.

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results with students. There will be presented some examples of situations in teaching that demonstrate both the possibility and urgency of following the objectives of "lower" levels for achieving the objectives of "higher" levels. Some destructive effects caused by the violation of this attitude will be pointed out as well.

Conference contributions

How University Students of Mathematics Evaluate the Correctness of a Mathematical Reasoning?

Marianna Ciosek and Anna K. Żeromska

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A conception to develop of university math students'(future teachers) understanding of mathematical methodology will be presented (Ciosek, Żeromska, 2013). The genesis of this concept are the results of research indicating students' difficulties to understand and construct mathematical proofs (eg, Gardiner, 1999; Jones, 2000; Ciosek, 2005; Żeromska, 2013). One of the reasons for these difficulties can be explained as follows. University students in the course of mathematics, very often learn mathematics as a "ready made knowledge". Perhaps because of the large scope of the learned content, prospective teachers have little opportunities to reflect on the fact that mathematics is not just a collection of definitions, properties and patterns, but it is also a specific, creative human activity.

Within the framework of mathematical studies an opportunity to engage in creative mathematical activity (in the sense of creating new subjective knowledge) can be elementary mathematics. In our concept we want to

Early Algebra in the Primary Schools

Noémi Székelyová

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Algebra is a key component of the primary school mathematics education. Its importance, especially as a gateway to high mathematics, is appreciated by education systems around the world. Algebra, as a branch of science, is present in schools not earlier than in the 8th grade of primary school. In the meantime students should be developing prerequisite skills necessary for a formal study of algebra such as understanding the language of algebra; understanding the concept of variable; and understanding the concepts of relation and function. In this contribution we give a brief look to the Slovak curriculum and compare it with US and UK curriculum. We also want to present some activities to develop the above mentioned skills and to promote higher levels of understanding. These will be supported by illustrative examples of students' works and solutions.

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Metric Properties of Chromatically-Critical Graphs

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For a given graph G we define $u(G)$ as the maximal number of edges of an unit-distance subgraph of G; for a graph family $\mathcal{G}$, we also define the number $r_u(n, \mathcal{G}) = \max\left\{\frac{u(G)}{|E(G)|} : G \in \mathcal{G}, |V(G)| = n\right\}$.

In connection with the Hadwiger-Nelson problem, a wide-spread belief is that if $\mathcal{G}$ is the class of all k-chromatic graphs with $k \geq 5$, then $r_u(n, \mathcal{G}) < 1$

We briefly survey the previous research on perfect matchings in bridgeless cubic graphs and discuss recent developments regarding 2-factors with special properties. In particular, an interesting problem consists of avoiding 5-cycles in a 2-factor; the best results known so far have been obtained by an algebraic approach. We explain how perfect matching polytopes provide a useful tool, discuss the limitations of our methods, and show an application of the obtained results in the problem of bounding circumference of cubic graphs.

Exotic Representations of Numbers

Edita Pelantová

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Several representations of numbers will be discussed, namely, the positional representations (with the positive, negative or complex bases and with the redundant digit sets) and also the modular representations. Properties of these representations with respect to arithmetic operations will be discussed. Applications will be mentioned concerning, i.a., the evaluation of elementary functions or the computer use of multiple parallel processors and/or of a large memory.

40th Anniversary of Classes Focused on Mathematics at Gymnasium Poštová Str. 9, Košice

Marián Ružička

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Gymnázium at Poštová Str. 9 in Košice, which has been located in a historical building in the centre of the city since its foundation in the academic year 1969/70, has been proud to be called "mathematical" for four decades. It was well-earned thanks to the quality and professional work of teachers and their students in the classes focused on mathematics, nowadays classes with extended teaching of mathematics. Students of the first mathematical

On the Total Thue Colourings of Graphs

Erika Škrabuľáková and Jens Schreyer

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Non-repetitive sequences were introduced to graph theory by Alon, Grytczuk, Hałuszcak and Riordan [1]. An *edge (vertex) k-colouring* of G is a mapping $\varphi : E(G) \to \{1, 2, \ldots, k\}$ ($\phi : V(G) \to \{1, 2, \ldots, k\}$). The edge-colouring φ (vertex-colouring ϕ) of a graph G is *non-repetitive* if the sequence of colours on every path in G is non-repetitive. (Here, by a repetition we mean such a sequence, in which there exist two consecutive identical blocks of terms. On the other hand, non-repetitive sequences are the sequences where no two consecutive blocks are the same.)

Here we deal with variants of non-repetitive total colourings of graphs. By a *weak total Thue colouring* of a graph G we mean a colouring Ψ of both vertices and edges of G such that a colour sequence on consecutive vertices and edges of every path in G is non-repetitive. Moreover, if both the induced vertex-colouring and the induced edge-colouring of G are non-repetitive under Ψ, we speak about a *(strong) total Thue colouring* of G. The minimum number of colours required in every weak total Thue colouring is called the *weak total Thue number* and it is denoted $\pi_{T_w}(G)$. The minimum number of colours required in every (strong) total Thue colouring is called the (strong) *total Thue number*, $\pi_T(G)$.

We proved that the (strong) total Thue number is smaller than $15 \cdot \Delta^2$, where $\Delta \geq 3$ is the maximum degree of the graph, and that the upper bound $18 \cdot \Delta^2$ holds for the list version of the problem. For the weak total Thue number we show that $\pi_{T_w}(G) \leq |E(G)| - |V(G)| + 5$, that gives $\pi_{T_w}(G) \leq 3 + k$ for planar graphs with k faces. We also give some upper and lower bounds for these parameters considering special classes of graphs.

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Cyclic Words and Edge Colourings of Plane Graphs (or How to Discover New Theorems)

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Let $\mathbb{A} = \{a, b, c, \dots\}$ be a finite alphabet, whose element are called letters (digits, colours, symbols, ...).

The *word* of length n over $\mathbb{A}$ is an expression $w = a_1 a_2 \dots a_n$, where $a_i \in \mathbb{A}$ for all $i = 1, 2, \dots, n$. *Subword* $\bar{w}$ of the word w is an expression $\bar{w} = a_i a_{i+1} \dots a_j$ with $1 \leq i \leq j \leq n$.

The *cyclic word* of length n is an expression $w = a_1 a_2 \dots a_n$, $n \geq 2$ (Consider the cyclic word as a sequence of consecutive labells on the vertices of a cycle of length). A subword of a cyclic word is its arbitrary part.

Let us recall some properties that words can have. A word is *proper* if it no two consecutive letters are the same. For example the word "*abcba*" is proper but the word "*abbcd*" is not proper.

The word $a_1 a_2 \dots a_n$, $n \geq 1$, is simple if $a_i \neq a_j$ for $i \neq j$.

The word of the form $a_1 a_2 \dots a_{2k}$ with property that $a_i = a_{i+k}$ for all $i = 1, 2, \dots, k$ is called the *repetition*. A word is called *nonrepetitive* is none of its subwords is a repetition.

Examples: The word "*abcabc*" is a repetition, the word "*abcacbd*" is nonrepetitive, while the word "*abcbcdea*" is not nonrepetitive because it contains a subword "*bcbc*", which is a repetition.

A *palindrom* is any word which can be read in the same way from the front and from the back. The word is *palindromfree* if no its subword is a palindrom

Examples: The words "*abcddcba*" and "*abcdcba*" are palindroms while the word *abcdbda* is palindrom free.

A word is a *weak parity* one if at least one letter in it appears there an odd number of times. A word is a *strong parity* one if each used letter in it is used there an odd numbers of times. For example the word "*abcabde*" is a weak parity word. The word "*abcabdadbd*" is a strong parity word.

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Quasi-Normal Convergence with Respect to Ideal

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H. Cartan [1] introduced the convergence of a function into a topological space with respect to ideal of subsets of its domain. If the function is a sequence of points in a topological space then we obtain convergence of this sequence with respect to ideal of sets of natural numbers. Such convergence as well as the corresponding pointwise convergence of a sequence of functions is now widely studied. P. Das and D. Chandra [2] introduced and initiated the investigation of ideal version of quasi-normal convergence of a sequence of functions.

We present the results related to ideal version of quasi-normal convergence obtained by R. Filipów and M. Staniszewski [3] and by us jointly with L. Bukovský and P. Das. Moreover, we present also our own results. The main one is the following simple characterization: For an ideal $\mathcal{J}$ on $\mathbb{N}$, all pointwise convergent sequences of real-valued functions converge quasi-normally with respect to $\mathcal{J}$ if and only if $\mathcal{J}$ contains an isomorphic copy of the ideal Fin $\times$ Fin on $\mathbb{N} \times \mathbb{N}$ defined by

$$\text{Fin} \times \text{Fin} = \{A \subseteq \mathbb{N} \times \mathbb{N};\ |\{n;\ |\{m;\ (n, m) \in A\}| = \aleph_0\}| < \aleph_0\}.$$

Variations on Antimagic Labelings

Andrea Feňovčíková

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A *labeling* or a *valuation* of a graph is any mapping that sends certain set of graph elements to a certain set of numbers, usually positive integers. If the domain of the labelings is the set of all the vertices we call such a labeling *vertex labelings*, while if only the edges of the graph are labeled we speak about *edge labelings*. If both vertices and edges of a graph are labeled, such a labeling is called *total*. Note that if we consider plane graphs then it is also possible to label the faces of a graph. In this case we call such labelings *face labelings*.

In many cases, it is interesting to consider the sum of all labels associated with a graph element. This will be called the *weight of an element*. The sums will be either a set of *vertex weights*, obtained for each vertex by adding all the labels of the vertex and its adjacent edges, or a set of *edge weights*, obtained for each edge by adding the labels of an edge and its end vertices, or a set of *face weights*, obtained for each face in a plane graph by adding all the labels of a face and vertices and edges surrounding that face.

One situation that is of interest is when all the weights are the same. In this case we speak about *labelings of magic type*. The natural extension of the notion of magic type labelings are *labelings of antimagic type*, when all weights are pairwise distinct.

In 1990, Hartsfield and Ringel [1] introduced the concept of an *antimagic labeling* of graph – an edge labeling where vertex-weights are requested to be just pairwise distinct. Hartsfield and Ringel [1] conjectured that every connected graph except K_2 has a vertex-antimagic labeling. This conjecture, and also its weaker version that every tree except K_2 admits a vertex-antimagic labeling, remain wide open. However, often it is very easy to find many different antimagic labelings for a given graph. Therefore, it is reasonable to investigate antimagic labelings with some restrictions placed on the weights. One well studied restriction is that the weights form an arithmetic progression with a common difference.

In this talk we will give a survey of known results on antimagic labeling and many variations of antimagic labelings will be discussed.

References

[1] Hartsfield, N., Ringel, G.: *Pearls in Graph Theory: A Comprehensive Introduction,* Academic Press Inc., Boston, MA, 1990.

- minimization of manual keyboard typing during the process of inout of publication data, when the publication is not indexed in Web of Science, Scopus, or other external database;
- follows the current categorization of publications and citations used by the Ministry of education (no. 456/2012 Z. z.) and leaves it unchanged;
- keeps and leaves unchanged the indexed structure of the Slovak universities and their faculties;
- allows efficient use of popular personal bibliography managers (JabRef, BibDesk, etc.).

During the workshop we will explain in detail the differences between the existing EPČ system and our LEPC system. We will demonstrate, how data from the Web of Science, Scopus and other external sources can be imported into LEPC; how further publications can be added incrementally; how citations lists can be added to the publications already registered in LEPC; how filtered data can be exported and then used in accordance with the needs of a user; and other practical aspects of working with LEPC. Workshop participants will have an opportunity to try LEPC themselves and to explore its possibilities and comfort *ad libitum*.

Fully functional public demo version of our LEPC system is currently available at the following address:
`http://omega.tuke.sk/student/patrik.zakutansky/lepc_demo/`

The introduced system in its generic form, i.e. without hard-linking it to the Slovak universities, can be used also as a general system for registration of technical documents and links between them. For this kind of applications our system allows defining an arbitrary categorization structure for the registered documents.

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Discrete Mathematics at FEE, CTU in Prague

Marie Demlova

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Five years ago, new study programs were accredited at the Faculty of Electrical Engineering, CTU in Prague. Previously there were only two study programs — Electrical Engineering and Informatics (bachelor and master program) and Software Technologies and Management (only bachelor program). Electrical Engineering and Informatics was substituted by four new programs — Electrical Engineering, Power Engineering and Management; Communication, Multimedia and Electronics; Cybernetics and Robotics; and Open Informatics. The last two programs are declared to be research-oriented. Naturally, it lead to a change in teaching mathematics, and especially discrete mathematics. The talk will focus on teaching discrete mathematics for student of the study program Open Informatics.

One of the concepts introduced in the program Open Informatics is that of a minor specialization. A student fulfills a minor (e.g. Mathematics minor, Computer Graphics minor, Artificial Intelligence minor, etc) if he/she successfully passes, within the scope of elective courses, at least 3 out of 4 obligatory courses for the minor in question. For Mathematics minor they are Advanced Calculus, Information Theory and Coding Theory, Mathematics for Cybernetics, and Graph Theory.

The talk will focus on experience with teaching discrete mathematics in both, obligatory and elective courses in the Mathematics minor.

In the second part of the talk couple of applications will be shown; applications that help to gain (and maintain) students' interest for concepts of discrete mathematics. These are RSA cryptosystem, Miller Rabin Primality test, and Huffman coding.

At the end of the talk a discussion is planned bringing experience of teaching discrete mathematics at different schools/universities.

Štvrtok – Thursday 3. 4. 2014

8^{30} – Jaroslav Šupina (ÚMV PF UPJŠ) *Quasi-Normal Convergence with Respect to Ideal*

8^{50} – Lukáš Štefanský (KMTI FEI TUKE) *Web Preferences Using Robust Fuzzy Discrete Dynamic Systems*

9^{10} – Miron Pavluš (FM PU) *Solution of the Moisture Transport Model in Porous Materials*

9^{30} – **Občerstvenie – Coffee-break**

10^{00} – Eva Potpinková (ÚMV PF UPJŠ) *Pareto Optimality in Many-to-Many Assignment Problems*

10^{20} – Jana Pócsová (ÚRaIVP FBERG TUKE) *About the Probability in the School Classroom*

10^{40} – Katarína Furcoňová (ÚMV PF UPJŠ) *The Use of Streaming Video to Support Teaching and Learning Math Word Problems*

11^{00} – Andrea Kanáliková (ÚMV PF UPJŠ) *The Issue of Irrational Numbers Teaching at High Schools*

12^{00} – **Obed – Lunch**

14^{10} – Jana Coroničová Hurajová (ÚMV PF UPJŠ) *On Decay Centrality in Graphs*

14^{30} – Peter Hudák (ÚMV PF UPJŠ) *Maximal Weight of Planar Graphs with Specified Density*

14^{50} – Tatiana Polláková (ÚMV PF UPJŠ) *On Supermagic Joins of Graphs*

15^{10} – Caroline Medina Graciano (México, KMTI FEI TUKE) *Optimal Drawings of* $K_{5,n}$

15^{30} – **Občerstvenie – Coffee-break**

16^{00} – Michaela Vrbjarová (ÚMV PF UPJŠ) *Unique Maximum Colourings of Plane Graphs*

16^{20} – Mária Kubíková (ÚMV PF UPJŠ) *Light Edges in Plane Graphs of Given Girth*

16^{40} – Pavol Široczki (ÚMV PF UPJŠ) *Metric Properties of Chromatically-Critical Graphs*

Sobota – Satursday 5. 4. 2014

8^{30} – Izabela Solarz (IM UP KEN, Kraków) *A Game Which Incidentally Teaches Algebra*

8^{50} – Maria Samborska (IM UP KEN, Kraków) *Engaging Middle School Students in Mathematics*

9^{10} – Marianna Ciosek and Anna K. Żeromska (IM UP KEN, Kraków) *How University Students of Mathematics Evaluate the Correctness of a Mathematical Reasoning?*

9^{30} – **Občerstvenie – Coffee-break**

10^{00} – Andrea Feňovčíková (KAMaI FMM TUKE) *Variations on Antimagic Labelings*

10^{45} – **Záver konferencie – Conference closing**

11^{00} – **Obed – Lunch**

Aj tento rok sa nám podarilo získať viacero výrazných osobností. Pozvanie prednášať prijali:

Bohdan Datsko, DrSc. (IAPMM NASU Lviv, Ukrajina),
prof. RNDr. Marie Demlová, CSc. (KM FE ČVUT Praha),
doc. RNDr. Jozef Fecenko, CSc. (KMaA FHI EU Bratislava),
doc. RNDr. Andrea Feňovčíková, PhD. (KAMaI SjF TU Košice),
doc. RNDr. Miroslav Haviar, CSc. (KM FPV UMB Banská Bystrica),
prof. RNDr. Stanislav Jendroľ, DrSc. (ÚMV PF UPJŠ Košice),
RNDr. Ján Mazák, PhD. (KMaI PF TU Trnava),
prof. Ing. Edita Pelantová, CSc. (KM FJFI ČVUT Praha),
RNDr. Marián Ružička (Gymnázium Poštová, Košice) a
RNDr. Ladislav Spišiak (Gymnázium Šrobárova, Košice).

Prajeme vám príjemný pobyt v Herľanoch

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