

**Jednota slovenských matematikov a fyzikov  
Pobočka Košice**

**Prírodovedecká fakulta UPJŠ  
Ústav matematických vied**

**Fakulta elektrotechniky a informatiky TU  
Katedra matematiky a teoretickej informatiky**

---

## **12. Konferencia košických matematikov**

---

Názov: 12. Konferencia košických matematikov

Editori: Ján Buša, Stanislav Jendroľ, Štefan Schrötter  
Vydala: Fakulta elektrotechniky a informatiky TU, Košice  
Vydanie: prvé  
Počet strán: 28  
Náklad: 60 ks  
Vydané v Košiciach, 2011  
Elektronická sadzba programom pdf $\TeX$

**ISBN 978-80-553-0658-2**

**Herľany  
13. – 16. apríla 2011**

- Juhás Matej** — Ústav matematických vied PF UPJŠ, Košice, SR,  
matej.juhás@student.upjs.sk
- Kanáliková Andrea** — Ústav matematických vied PF UPJŠ, Košice, SR,  
andrea.kanalikova@student.upjs.sk
- Klešč Marián** — Katedra matematiky FEI TU, Košice, SR,  
Marian.Klesc@tuke.sk
- Knor Martin** — Katedra matematiky SvF STU, Bratislava, SR,  
knor@math.sk
- Kopperová Mária** — Ústav matematických vied PF UPJŠ, Košice, SR,  
maria.kopperova@student.upjs.sk
- Körtesi Péter** — Institute of Mathematics, University of Miskolcs,  
Hungary, matkp@uni-miskolc.hu
- Krajník Filip** — Ústav matematických vied PF UPJŠ, Košice, SR,  
filip.krajnik@student.upjs.sk
- Mockovčiaková Martina** — ÚMV PF UPJŠ Košice, SR,  
martina.mockovciakova@student.upjs.sk
- Myšková Helena** — Katedra matematiky a teoretickej informatiky FEI  
TU Košice, SR, helena.myskova@tuke.sk
- Petrillová Jana** — Katedra matematiky a teoretickej informatiky FEI  
TU Košice, SR, jana.petrillova@tuke.sk
- Pillárová Eva** — Ústav matematických vied PF UPJŠ, Košice, SR,  
eva.pillarova@student.upjs.sk
- Pócs Jozef** — Matematický ústav SAV, Košice, SR, pocs@saske.sk
- Pócsová Jana** — Ústav riadenia a informatizácie výrobných procesov  
BERG TU, Košice, SR, jana.pocsova@tuke.sk
- Polláková Tatiana** — Ústav matematických vied PF UPJŠ, Košice, SR,  
tatiana.pollakova@student.upjs.sk
- Repiský Michal** — Ústav matematických vied PF UPJŠ, Košice, SR,  
michal.repisky@student.upjs.sk

## Predhovor

Milí priatelia,

vítame Vás na 12. Konferencii košických matematikov. Túto konferenciu organizuje Jednota slovenských matematikov a fyzikov, pobočka Košice, v spolupráci s Ústavom matematických vied Prírodovedeckej fakulty UPJŠ, Centrom excelentnosti inforatických vied a znalostných systémov UPJŠ, katedrami matematiky Technickej univerzity a pobočkou Slovenskej spoločnosti aplikovanej kybernetiky a informatiky pri KRVP BF TU v Košiciach. Konferencia sa koná, tak ako aj jej predchádzajúce ročníky, v útulnom prostredí Učebno-výcvikového zariadenia TU Košice v Herľanoch.

Cieľom konferencie je zintenzívniť stavovský život všetkých, ktorí sa v Košiciach a okolí profesionálne zaoberajú matematikou (t. j. učiteľov všetkých typov škôl, pracovníkov na poli matematických a inforatických vied a aplikácií matematiky v priemysle, technike, bankovníctve a inde) a formulovať základné oblasti ich stavovských záujmov. Odborný program konferencie tradične pozostáva z pozvaných prednášok, prihlásených referátov a diskusií o stavovských problémoch. Prvé dva dni sú venované prezentácii výsledkov mladých matematikov a doktorandov. Piatkový a sobotňajší program napĺňajú prednášky pozvaných prednášateľov a prihlásené príspevky. V programe konferencie je vytvorený priestor aj na diskusiu o aktuálnych problémoch.

Táto štruktúra programu sa vyprofilovala z poznatkov minulých ročníkov konferencie. Doktorandom a mladším matematikom je poskytnutý priestor na získanie skúseností pri prezentácii svojich výsledkov. Je potešujúce vidieť, ako sa každým rokom zlepšujú ich vystúpenia. Veríme, že im vystúpenia na tejto konferencii pomôžu pri prezentovaní výsledkov na ďalších konferenciách. Organizačný výbor konferencie sa snaží pozývať významné osobnosti matematiky, ktoré v rámci svojich prednášok ukážu miesto matematiky v spoločenskom živote a súčasné trendy jej rozvoja. Nejedna z pozvaných prednášok mala taký pozitívny ohlas, že ich autori boli pozvaní predniesť ich aj na iných konferenciách.

Toho roku pozvanie prednášať prijali: RNDr. B. Baculíková, PhD. (FEI TU), mim. prof. RNDr. V. Bálint, CSc. (FPEDaSU ŽU Žilina), prof. RNDr. L. Bukovský, DrSc. (PF UPJŠ) RNDr. J. Hnatová, PhD. (MPC Prešov), doc. RNDr. J. Ivančo, CSc. (PF UPJŠ), prof. RNDr. M. Knor, Dr. (SvF STU Bratislava), Dr. P. Körtesi (IoM U Miskolcs, Maďarsko), RNDr. H. Myšková, PhD. (FEI TU) a RNDr. J. Pócs, PhD. (MÚ SAV Košice).

Prajeme Vám príjemný pobyt v Herľanoch

Organizačný výbor:  
Stanislav Jendroľ  
Ján Buša  
Štefan Schrötter

14<sup>50</sup> – Körtesi P. (IoM MU) *Isogonal Transformations – Revisited with GeoGebra*

15<sup>40</sup> – **Káva – Coffee-break**

16<sup>10</sup> – Hnatová J. (Prešov) *Mathematics Maturity Exam*

17<sup>00</sup> – Bálint V. (ŽU) *A Look into the Inside of IMO*

18<sup>00</sup> – **Večera – Dinner**

19<sup>00</sup> – **Spoločenský večer – Party**

### Sobota – Saturday 16. 4. 2011

8<sup>30</sup> – Škrabuľáková E. (KAMA Sjf) *Facial Thue Choice Index of Semiregular Polyhedra*

8<sup>55</sup> – Buša J. (KMTI FEI) *Teaching Mathematics with Inspiration and Dedication*

9<sup>15</sup> – Baculíková B. (KMTI FEI) *Oscillatory Properties of Third Order Differential Equations with Mixed Arguments*

10<sup>00</sup> – **Káva – Coffee-break**

10<sup>15</sup> – Pócs J. (MÚ SAV) *Convexities of Lattices*

11<sup>00</sup> – **Záver – Conclusion**

11<sup>10</sup> – **Obed – Lunch**

Rusnačko R. *The Growth Curve Model with Uniform Correlation Structure*..... 19

Škrabuľáková E. *Facial Thue Choice Index of Semiregular Polyhedra* .. 20

Šupina J. *SSP-Property and Similar Principles* ..... 21

Vojtová M. *A Note on Deterministic Population Dynamics Models* .... 21

**Program konferencie – Conference programme** ..... 22

**Zoznam účastníkov – List of participants** ..... 25

**Program 12. Konferencie košických matematikov**  
**Programme**  
**of the 12th Conference of Košice Mathematicians**

**Streda – Wednesday 13. 4. 2011**

14<sup>45</sup> – **Otvorenie – Opening**

14<sup>50</sup> – Polláková (ÚMV UPJŠ) *Supermagic Non-Regular Graphs*

15<sup>15</sup> – Šupina J. (ÚMV UPJŠ) *SSP-Property and Similar Principles*

15<sup>40</sup> – Ivanová V. (ÚMV UPJŠ) *Metric Preserving Functions*

16<sup>05</sup> – **Káva – Coffee-break**

16<sup>30</sup> – Kopperová M. (ÚMV UPJŠ) *On Vertices Enforcing a Hamiltonian Cycle*

16<sup>55</sup> – Juhás M. (ÚMV UPJŠ) *Characterization of Standard Extreme Value Distributions Using Records*

17<sup>20</sup> – Pillárová E. (ÚMV UPJŠ) *A Near Equitable Cake-Cutting Algorithm for Three Players*

18<sup>00</sup> – **Večera – Dinner**

**Štvrtok – Thursday 14. 4. 2011**

9<sup>00</sup> – Farkasová Z. (ÚMV UPJŠ) *Radical Classes*

9<sup>25</sup> – Repiský M. (ÚMV UPJŠ) *Stability in Housing Markets*

9<sup>50</sup> – Rusnačko R. (ÚMV UPJŠ) *The Growth Curve Model with Uniform Correlation Structure*

10<sup>15</sup> – **Káva – Coffee-break**

10<sup>45</sup> – Šugerek P. (ÚMV UPJŠ) *Parity Vertex Colouring of Semiregular Graphs*

11<sup>10</sup> – Hudák D. (ÚMV UPJŠ) *Light Edges in 1-Planar Graphs with Prescribed Minimum Degree*

## On the History of the Concept of Real Number

Lev Bukovský

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

In the history of investigation of the real number, we distinguish four periods:

- 1) Understanding the real numbers system beginning about 2 000 B.C.
- 2) Exact definitions of the real numbers system in the half of the 19. century.
- 3) The third period – the end of 19. century till sixties of 20. century.
- 4) The invention of forcing in sixties as a tool to show that many questions raised in the set theory are undecidable and as a consequence, one cannot answers many questions about the real number system in the framework of the set theory.

The lecture will be concentrated mainly on the first and the second periods, presenting some examples of “unanswerable” questions.

Finally I will shortly say about the rigorization of infinitesimal calculus as established by Newton, Leibnitz and mainly Euler by the methods of mathematical logic (nonstandard models) in the second half of 20. century.

## Facial Thue Choice Index of Semiregular Polyhedra

Erika Škrabuľáková

Department of Applied Mathematics FME, Technical University,  
Letná 9, 040 01 Košice, Slovakia

A *polyhedron*  $P$  in the three-dimensional Euclidean space is a finite collection of planar convex polygons, called the *faces*, such that every edge of every polygon is an edge of precisely one other polygon. The edge set of a polyhedron is the set of intersections of adjacent faces, and the vertex set is the set of intersections of adjacent edges. A polyhedron  $P$  is called *semiregular* if all of its faces are regular polygons and there exists a sequence  $\sigma = (p_1, p_2, \dots, p_q)$ , such that every vertex of  $P$  is surrounded by a  $p_1$ -gon, a  $p_2$ -gon,  $\dots$ , a  $p_q$ -gon, in this order within rotation and reflexion. If  $G$  is a plane graph, a *facial non-repetitive edge-colouring* of  $G$  is an edge-colouring such that any *facial trail* is non-repetitive. Moreover if the colour of every edge  $e$  is chosen only from pre-assigned list of colours  $L(e)$  we speak about a *facial non-repetitive list edge-colouring*  $\varphi_{fl}$  of the graph  $G$ , where  $L : E(G) \rightarrow 2^{\mathbb{N}}$  is a list assignment of  $G$ . The minimum list length needed is the *facial Thue choice index* of  $G$  and it is denoted by  $\pi'_{fl}(G)$ . For the graph  $G$  from the family of semiregular polyhedra we show  $\pi'_{fl}(G) \leq 7$  except of a single graph where  $\pi'_{fl}(G) \leq 8$ . Moreover we show that for infinitely many graphs the bound 7 should be reduced to 5.

## Literatúra

- [1] J. Grytczuk, J. Przybyło, X. Zhu, *Nonrepetitive list colourings of paths*, Random Structures & Algorithm, Vol. 38, Issue 1-2, January-March 2011, (2011), 162-173.
- [2] J. Schreyer, E. Škrabuľáková, *Upper bounds for facial Thue choice index of plane graphs*, manuscript (2011).

## On Magic Graphs

Jaroslav Ivančo

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

A graph is called *magic* if it admits a labelling of the edges by pairwise different positive integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. Magic graphs extend the classic concept of magic squares. They were introduced by J. Sedláček in 1963. In the talk we map the development of magic topic from its beginning up to now. We survey main results and methods of constructions of magic graphs. We describe some extension of the topic and also present several open problems.

## Isogonal Transformations — Revisited with GeoGebra

Péter Körtesi

Institute of Mathematics, University of Miskolc, Hungary

The symmedian lines and the symmedian point of a given triangle present interesting properties. Part of these properties can be formulated in a more general context for isogonals.

In a triangle the isogonal of a line passing through one of the vertices of the triangle is a line symmetric to the bisector of the given angle. It can be proven that the three isogonals of three concurrent lines which pass through the three vertices of the triangle, are concurrent. This property serves as definition for the isogonal transformation, the image of a given point in this transformation will be the intersection point of the three isogonals of the three lines which pass through the given point and the vertices of the triangle. The lecture is aimed to present some of the properties of the isogonal transformations, and to visualise them using GeoGebra.

## The Crossing Number of Cartesian Product of 6-Vertex Graphs with Paths

Jana Petrillová

Department of Mathematics and Theoretical Informatics FEI,  
Technical University,  
B. Nemcovej 32, 040 01 Košice, Slovakia

The investigation on the crossing numbers of graphs is a classical and however very difficult problem. Let  $G_1$  and  $G_2$  be simple graphs with vertex sets  $V(G_1)$  and  $V(G_2)$ , and edge sets  $E(G_1)$  and  $E(G_2)$ . The Cartesian product  $G_1 \times G_2$  of graphs  $G_1$  and  $G_2$  has vertex set  $V(G_1 \times G_2) = V(G_1) \times V(G_2)$  and any two vertices  $(u, u')$  and  $(v, v')$  are adjacent in  $G_1 \times G_2$  if and only if either  $u = v$  and  $u'$  is adjacent with  $v'$  in  $G_2$ , or  $u' = v'$  and  $u$  is adjacent with  $v$  in  $G_1$ . The aim of this presentation is to find the crossing numbers of the Cartesian product of paths with some graphs of order six. The similar problem was solved for the Cartesian product of cycles with 6-vertex trees and also for the Cartesian product of paths, cycles and stars with some graphs of order four and five. In this presentation, we give first the crossing number of the Cartesian product of the special graph  $H$  on six vertices with the path  $P_n$  on  $n$  edges. Then we use this result to determine crossing numbers of Cartesian product of another 6-vertex graphs with the path.

## Supermagic Non-Regular Graphs

Tatiana Polláková

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

A graph is called magic (supermagic) if it admits a labeling of the edges by pairwise different (and consecutive) integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. We will deal with magic and supermagic joins of graphs and we will establish some conditions for magic joins of graphs to be supermagic.

## Conference contributions

### Balanced Degree-Magic Complements of Bipartite Graphs

Ľudmila Bezegová

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

A graph is called supermagic if it admits a labelling of the edges by pairwise different consecutive positive integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. A graph is called *degree-magic* if it admits a labelling of the edges by integers  $1, 2, \dots, |E(G)|$  such that the sum of the labels of the edges incident with any vertex  $v$  is equal to  $\frac{1+|E(G)|}{2} \deg(v)$ . A *degree-magic labelling* of a graph is a generalized supermagic labelling of regular graphs. We used a degree-magic labelling for construction of supermagic graphs. Construction of some balanced degree-magic complements of bipartite graphs are presented.

### Teaching Mathematics with Inspiration and Dedication

Ján Buša

Department of Mathematics and Theoretical Informatics FEI,  
Technical University,  
B. Nemcovej 32, 040 01 Košice, Slovakia

A short information about the study visit in the framework of CEDE-FOP (European Centre or Development of Vocational Training) Lifelong Learning Programme is presented.

## Understanding of the Rational Number Notion by First Year Undergraduates

**Andrea Kanáliková and Jana Pócsová**

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia  
and Institute of Control and Informatization of Production  
Processes, BERG TU, B. Němcovej 3, 040 01 Košice, Slovakia

A substantial attention is devoted to the rational number notion in mathematics teaching nowadays. Even though, the knowledge of students does not correspond to that reality. This contribution deals with the analysis of results obtained from a pre-test focused to the profound of rational number understanding by first class undergraduates. The contribution also includes results of particular tasks which through the ability of students to sort and locate irrational numbers on the number line, tending of students to modification of rational numbers to the decimal or fraction form and to application of rational number notion definition, were investigated.

## On Vertices Enforcing a Hamiltonian Cycle

**Mária Kopperová**

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

A nonempty vertex set  $X \subseteq V(G)$  of a hamiltonian graph  $G$  is called an *H-force* set of  $G$  if every *X-cycle* of  $G$  (i.e. a cycle of  $G$  containing all vertices of  $X$ ) is hamiltonian. The *H-force* number  $h(G)$  of a graph  $G$  is defined to be the smallest cardinality of an *H-force* set of  $G$ . We established exact values of this parameter for two classes of graphs, namely generalized dodecahedra and circulant graphs.

can be found using the Top Trading Cycles algorithm attributed to Gale, although this algorithm cannot output all core allocations in the general case. In this talk we will describe the structure of the core in some simple markets, derived from geometric representation of the agents, where agents are located in equidistant points on the line or in the vertices of a regular polygon. We will show a connection between the number of core allocations and some well-known integer sequences such as the Fibonacci sequence or Catalan numbers. We will also show that it is NP-hard to decide whether the core of a general housing market contains an allocation with some special property.

## Radical Classes

**Zuzana Farkasová and Danica Jakubíková-Studenovská**

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

The notions of a radical class and of a torsion class were investigated for lattice ordered groups. We deal with monounary algebras. First, it is proved that a class of partial monounary algebras is a radical class iff it is a torsion class. Also, we describe all radical classes of partial monounary algebras. Moreover, we define a homog-radical class of monounary algebras and characterize all homog-radical classes of monounary algebras.

## Light Edges in 1-Planar Graphs with Prescribed Minimum Degree

Dávid Hudák and Peter Šugerek

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

According to the famous theorem of A. Kotzig on light edges in 3-connected planar graphs we investigate light edges in certain nonplanar graphs which can be drawn in the plane in such a way that each edge is crossed by at most one other edge; such graphs are called 1-planar. We prove that each 1-planar graph of minimum degree  $\delta \geq 4$  contains an edge with degrees its endvertices of type  $(4, \leq 13)$  or  $(5, \leq 9)$  or  $(6, \leq 8)$  or  $(7, 7)$ . We also show that for  $\delta \geq 5$  are these bounds best possible and that the list of edge types is minimal.

**Key words:** 1-planar graph, light edge

## Metric Preserving Functions

Veronika Ivanová

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

We call a function  $f : R^+ \rightarrow R^+$  metric preserving iff  $f(d) : M \times M \rightarrow R^+$  is a metric for every metric  $d : M \times M \rightarrow R^+$ , where  $(M, d)$  is an arbitrary metric space and  $R^+$  denotes the set of nonnegative reals. If  $f, g : R^+ \rightarrow R^+$ , then their infimal convolute  $f \square g$  is the following function:  $(f \square g)(x) = \inf\{f(y) + g(z) : y, z \in R^+ \text{ and } y + z = x\}$  for  $\forall x \in R^+$ . With some examples we introduce this topic to the reader. We provide that class of all metric preserving functions is not closed in the sense of infimal convolution. Also we introduce some interesting results of metric preserving functions in Math science and also in Computer Science.

## Parity Vertex Colouring of Semiregular Graphs

Stanislav Jendroľ and Peter Šugerek

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

A proper vertex colouring of a 2-connected plane graph  $G$  is a parity vertex colouring if for each face  $f$  and each colour  $c$ , either no vertex or an odd number of vertices incident with  $f$  is coloured with  $c$ . The minimum number of colours used in such a colouring of  $G$  is called parity chromatic number and denoted by  $\chi_s(G)$ .

We determine the  $\chi_s(G)$  of the Platonic solids graphs, the Archimedean solids graphs and some its extensions.

**Key words:** parity colouring

## Characterization of Standard Extreme Value Distributions Using Records

Matej Juhás and Valéria Skřivánková

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

A sequence  $\{X_n, n \geq 1\}$  of independent identically distributed random variables with common absolutely continuous distribution function  $F(x)$  and probability density function  $f(x)$  is considered. Random variable  $X_n$  is a lower record if  $X_n < \min\{X_1, X_2, \dots, X_{n-1}\}$ . By convention  $X_1$  is a lower record value. Let  $\{T_n, n \geq 1\}$  be the lower record times at which record values occur. We consider discrete time and define  $T_1 = 1$  and  $T_n = \min\{i; i > T_{n-1}, X_i < X_{T_{n-1}}\}$ . So the sequence  $\{L_n, n \geq 1\} = \{X_{T_n}, n \geq 1\}$  is a sequence of lower record values.

The contributions deals with characterization of standard extreme value distributions. Some characterizations by the independence of upper record values are known. We present criterions using the independence of some suitable function of lower record in a sequence of independent identically distributed random variables  $\{X_n, n \geq 1\}$ . We prove that  $X$  has standard Gumbel distribution if and only if  $e^{-L_m}$  and  $e^{-L_n} - e^{-L_m}$  are independent for  $1 \leq m < n$  and  $X$  has standard Fréchet distribution if and only if  $L_m^{-\alpha}$  and  $L_n^{-\alpha} - L_m^{-\alpha}$  are independent for  $1 \leq m < n, \alpha > 0$ .



## A Near Equitable Cake-Cutting Algorithm for Three Players

Katarína Cechlárová and Eva Pillárová

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

**Keywords:** cake cutting, algorithm, approximation

The cake represents a heterogeneous, infinitely divisible good that has to be divided among  $n$  players. Players may have different values for parts of the cake. Each player must get what he perceives to be a fair share. However, the notion of fairness can be formalized in several different ways. We considered the so-called equitability, i.e. all players have to be assigned pieces with exactly the same values according to their evaluation. We will show that such divisions exist for each player's order even if we insist that each player receives exactly one interval. We will prove that there is no finite algorithm that finds an equitable cake division with connected pieces for three players. Our finite algorithm finds a near equitable division with connected pieces using only evaluation and cutting queries.

## Core of Geometric Housing Markets

Katarína Cechlárová and Michal Repiský

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

The model of housing markets was introduced in 1974 by Shapley and Scarf. In a housing market there is a finite set of agents, each one owns one unit of a unique indivisible good (called a house) and wants to exchange it for another, more preferred one. The outcome of the exchange of the houses is called an allocation. The core of a housing market consists of allocations such that no coalition of agents may, by suitably rearranging their houses, make all its members better off compared to the allocation in question. It is known that each housing market has a nonempty core and a core allocation

## Intersection of Compact Congruences of Algebras

Filip Krajník

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

We investigate the intersection of two compact congruences. For a locally finite and congruence distributive variety  $\mathcal{V}$  we show necessary and sufficient condition of compactness of intersection of two compact congruences of  $A \in \mathcal{V}$ . We apply the general result to some varieties.

## Vertex-Distinguishing Edge Colorings of Circulant Graphs

Martina Mockovčiaková

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

Let  $\varphi : E \rightarrow \{1, 2, \dots, k\}$  be a proper edge coloring of a graph  $G = (V, E)$ . The set of colors of the incident edges of a vertex  $u \in V$  is called the *color set* of vertex  $u$  and is denoted  $S(u)$ . A coloring  $\varphi$  is *vertex-distinguishing* if  $S(u) \neq S(v)$  for any two distinct vertices  $u$  and  $v$  of  $G$ . A *strong chromatic index*  $\chi'_s(G)$  is the minimum number of colors of such coloring.

A *d-strong edge coloring* of  $G$  is a proper edge coloring that distinguishes any two distinct vertices  $u$  and  $v$  with distance  $d(u, v) \leq d$ . The minimum number of colors of *d-strong edge coloring* of graph  $G$  is called *d-strong chromatic index*  $\chi'_d(G)$ .

We present some general results on these chromatic indices of circulant graphs  $C_n(1, 2)$  and we give exact values for  $d = 1$  and  $d = 2$ .

**Keywords:** vertex-distinguishing coloring, circulant graph

## Solvability of Interval System of Linear Equations in (Max,Plus/Min) Algebras

Helena Myšková

Department of Mathematics and Theoretical Informatics FEI,  
Technical University,  
B. Nemcovej 32, 040 01 Košice, Slovakia

We shall deal with the solvability of interval systems of linear equations in the max-plus and max-min algebra. The max-plus and max-min algebra are the algebraic structures in which classical addition and multiplication are replaced by  $\oplus$  and  $\otimes$ , where  $a \oplus b = \max\{a, b\}$ ,  $a \otimes b = a + b$  in the max-plus algebra and  $a \oplus b = \max\{a, b\}$ ,  $a \otimes b = \min\{a, b\}$  in the max-min algebra.

The notation  $\mathbf{A} \otimes x = \mathbf{b}$  represents an interval system of linear equations, where  $\mathbf{A} = \langle \underline{A}, \overline{A} \rangle$  and  $\mathbf{b} = \langle \underline{b}, \overline{b} \rangle$  are given matrix interval and vector interval, respectively. We can define several types of solvability of interval systems of linear equations. We show how is the T4 solvability related to the properties of the sequence of matrix powers in the max-plus algebra. We give necessary and sufficient conditions for particular solvability concepts and describe the structure of the set of all solvability concepts in the max-plus and max-min algebra.

## Convexities of Lattices

Jozef Pócs

Mathematical Institute, Slovak Academy of Sciences, Grešákova  
6, 040 01 Košice, Slovakia

The notion of convexity of lattices has been introduced by E. Fried as analogy to the notion of variety of algebraic structures. A class  $\mathcal{C}$  of lattices is said to be a convexity, whenever  $\mathcal{C}$  is closed under homomorphic images, convex sublattices and direct products. System of all convexities of lattices partially ordered by the class-theoretical inclusion forms a complete lattice (omitting the fact that this system does not form a set). We describe some properties of this lattice and also discuss some open problems proposed by several authors.

## The Growth Curve Model with Uniform Correlation Structure

Rastislav Rusnačko

Institute of Mathematics FSc, Pavol Jozef Šafárik University,  
Jesenná 5, 041 54 Košice, Slovakia

The growth curve model is a generalized multivariate analysis of variance model, which is a useful statistical model for various areas of study including economics, agriculture, psychology, medicine or biology. This model is defined as

$$Y = XBZ + \varepsilon, \quad E\varepsilon = 0, \quad \text{var}(\text{vec } \varepsilon) = \Sigma \otimes I,$$

where  $Y_{n \times p}$  is matrix of observations,  $X_{n \times m}$  is ANOVA matrix,  $B_{m \times r}$  is matrix of unknown parameters,  $Z_{r \times p}$  is matrix of regress constants,  $\varepsilon_{n \times p}$  is matrix of random errors and  $\Sigma_{p \times p}$  is variance matrix of rows of matrix  $Y$ . There are a many special cases of this model in dependence on covariance structure. We deal with the two most commonly used structures namely

- Uniform covariance structure:  $\Sigma = \sigma^2[(1 - \rho)I + \rho \mathbf{1}\mathbf{1}']$ ,  
where  $\sigma > 0$  and  $\rho \in \left(-\frac{1}{p-1}, 1\right)$  are unknown parameters.
- Generalized uniform correlation structure:  $\Sigma = \theta_1 G + \theta_2 w w'$ ,  
where  $G$  is known symmetric matrix,  $w$  is known vector and  $\theta_1$  and  $\theta_2$  are called variance parameters.

## From Heawood conjecture to $n^{an^2}$ triangulations by complete graphs

Mike Grannell and Martin Knor

Department of Mathematics FCE, Slovak University of Technology Bratislava, Radlinského 11, 813 68 Bratislava, Slovakia

In 1890 Heawood conjectured the minimum number of colours that is sufficient to colour properly every graph embedded into an (orientable) closed and compact surface of non-positive Euler characteristics. This conjecture was proved in 1968 by Ringel and Youngs who found, for every admissible  $n$ , one triangulation by  $K_n$ . Later it was proved that there exist many more such triangulations. In the talk we discuss techniques, using which we recently proved that there are  $n^{an^2}$  triangulations by  $K_n$  for linear classes of values  $n$  (in the case of non-orientable embeddings) and “almost linear” classes of values  $n$  (in the case of orientable embeddings). We concentrate ourselves to orientable embeddings. Trivial upper bound for the number of triangulations by  $K_n$  is  $n^{n^2/3}$ .

## Mathematics Graduation Exam

Jana Hnatová

Methodical-pedagogical center, Tarasa Ševcenka 11, Prešov, Slovakia

Maturity exam is the official certificated exam held in the whole Slovakia and it is taken in the final year of the higher secondary education. In our contribution we deal with the history, current problems and upcoming changes in the implementation of school-leaving examination at us. We will analyse problems in detail (general, legislative, content, formal, pedagogical and psychological) have been occurred in the tasks of maturita assignments in mathematics.

## SSP-Property and Similar Principles

Jaroslav Šupina

Institute of Mathematics FSc, Pavol Jozef Šafárik University, Jesenná 5, 041 54 Košice, Slovakia

A topological space  $X$  has the SSP-property if whenever there are sequences  $A_n, n \in \mathbb{N}$  of real valued functions converging to zero on  $X$ , there is a sequence converging to zero which contains one function from each  $A_n$ . Modification of this property, QSQ-principle, contributed to alternative proofs of Tsaban – Zdomskyy’s Theorem and Reclaw’s Theorem. We introduce SSP and we present known relations among its modifications similar to QSQ.

## A Note on Deterministic Population Dynamics Models

Michaela Vojtová

Department of Mathematics and Statistics, Faculty of Science, Masaryk University, Kotlářská 2, 611 37 Brno, Czech Republic

In this talk we study sufficient conditions on Lotka–Volterra systems

$$\dot{x}_j = \varepsilon_j x_j + \sum_{k=1}^n a_{jk} x_j x_k \quad \text{for } j = 1, \dots, n \quad (1)$$

to admit conservation law and we discuss aspects of the Hamiltonian structure of systems of type (1). We show that for stable dissipative Lotka–Volterra systems with a singular point  $\mathbf{q} \in \text{Int}(\mathbb{R}_+^n)$  the dynamics on the attractor are Hamiltonian.

## Invited lectures

### Oscillatory Properties of Third Order Differential Equations with Mixed Arguments

**Blanka Baculíková**

Department of Mathematics and Theoretical Informatics FEI,  
Technical University,  
B. Nemcovej 32, 040 01 Košice, Slovakia

We present sufficient conditions for property B and the oscillation of the third-order nonlinear functional differential equation with mixed arguments

$$[a(t) [x'(t)]^\gamma]'' = q(t)f(x[\tau(t)]) + p(t)h(x[\sigma(t)]),$$

where  $\int^\infty a^{-1/\gamma}(s) ds = \infty$ . We deduce properties of the studied equations by establishing new comparison theorems so that property B and the oscillation is resulted from the oscillation of suitable first order equations.

### A Look into the Inside of IMO

**Vojtech Bálint**

Department of Quantitative Methods and Economic  
Informatics, University of Žilina, Univerzitná 1, 01026 Žilina,  
Slovakia

International Mathematical Olympiad (IMO) is a competition for secondary school pupils and usually lasts 12 days. Over 500 students from more than 100 countries attend this competition in recent years. Therefore it is clear that the IMO is a highly administrative challenge. In the lecture will be in more detail described the running of the IMO, here I mention only areas: Jury, Shortlist and selecting of problems, translation – linguistic curiosities and mutual control, competition – pupils' questions, repair of solutions, strategy for the coordination, coordination and scoring, setting boundaries for success and medals.

11<sup>35</sup> – Kanáliková (ÚMV UPJŠ) *Understanding of the Rational Number Notion by First Year Undergraduates*

12<sup>00</sup> – **Obed – Lunch**

15<sup>15</sup> – Petrillová J. (KMTI FEI TU) *The Crossing Number of Cartesian Product of 6-Vertex Graphs with Paths*

15<sup>40</sup> – Vojtová M. (Brno) *A Note on Deterministic Population Dynamics Models*

16<sup>05</sup> – Mockovčiaková M. (ÚMV UPJŠ) *Vertex-Distinguishing Edge Colorings of Circulant Graphs*

16<sup>30</sup> – **Káva – Coffee-break**

17<sup>00</sup> – Krajník F. (ÚMV UPJŠ) *Intersection of Compact Congruences of Algebras*

17<sup>25</sup> – Bezegová Ľ. (ÚMV UPJŠ) *Balanced Degree-Magic Complements of Bipartite Graphs*

18<sup>00</sup> – **Večera – Dinner**

**Piatok – Friday 15. 4. 2011**

8<sup>30</sup> – Myšková H. (KMTI FEI TU) *Solvability of Interval System of Linear Equations in (max,plus/min) Algebras*

9<sup>20</sup> – Bukovský L. (ÚMV UPJŠ) *On the History of the Concept of Real Number*

10<sup>30</sup> – **Káva – Coffee-break**

11<sup>00</sup> – Ivančo J. (ÚMV UPJŠ) *On Magic Graphs*

12<sup>00</sup> – **Obed – Lunch**

14<sup>00</sup> – Knor M. (CEF SU) *From Heawood Conjecture to  $n^{an^2}$  Triangulations by Complete Graph*

## Obsah – Contents

Predhovor – Preface ..... 3

## Pozvané prednášky – Invited lectures

Baculíková B. <i>Oscillatory Properties of Third Order Differential Equations with Mixed Arguments</i> .....	6
Bálint V. <i>A Look into the Inside of IMO</i> .....	6
Bukovský L. <i>On the History of the Concept of Real Number</i> .....	7
Grannell M. and Knor M. <i>From Heawood Conjecture to <math>n^{an^2}</math> Triangulations by Complete Graph</i> .....	8
Hnatová J. <i>Mathematics Maturity Exam</i> .....	8
Ivančo J. <i>On Magic Graphs</i> .....	9
Körtesi P. <i>Isogonal Transformations – Revisited with GeoGebra</i> .....	9
Myšková H. <i>Solvability of Interval System of Linear Equations in (max,plus/min) Algebras</i> .....	10
Pócs J. <i>Convexities of Lattices</i> .....	10

## Konferenčné príspevky – Conference contributions

Bezegová Ľ. <i>Balanced Degree-Magic Complements of Bipartite Graphs</i> ..	11
Buša J. <i>Teaching Mathematics with Inspiration and Dedication</i> .....	11
Cechlárová K. and Pillárová E. <i>A Near Equitable Cake-Cutting Algorithm for Three Players</i> .....	12
Cechlárová K. and Repiský M. <i>Stability in Housing Markets</i> .....	12
Farkasová Z. and Jakubíková-Studenovská D. <i>Radical Classes</i> .....	13
Hudák D. <i>Light Edges in 1-Planar Graphs with Prescribed Minimum Degree</i> .....	14
Ivanová V. <i>Metric Preserving Functions</i> .....	14
Jendroľ S. and Šugerek P. <i>Parity Vertex Colouring of Semiregular Graphs</i> ..	15
Juhás M. and Skřivánková V. <i>Characterization of Standard Extreme Value Distributions Using Records</i> .....	15
Kanáliková A. and Pócsová J. <i>Understanding of the Rational Number Notion by First Year Undergraduates</i> .....	16
Kopperová M. <i>On Vertices Enforcing a Hamiltonian Cycle</i> .....	16
Krajník F. <i>Intersection of Compact Congruences of Algebras</i> .....	17
Mockovčiaková M. <i>Vertex-Distinguishing Edge Colorings of Circulant Graphs</i> .....	17
Petrillová J. <i>The Crossing Number of Cartesian Product of 6-Vertex Graphs with Paths</i> .....	18
Polláková T. <i>Supermagic Non-Regular Graphs</i> .....	18

## Zoznam účastníkov – List of participants

- Bača Martin** — Katedra aplikovanej matematiky Sjf TU, Košice, SR, martin.baca@tuke.sk
- Baculíková Blanka** — Katedra matematiky a teoretickej informatiky FEI TU, Košice, SR, Blanka.Baculikova@tuke.sk
- Bálint Vojtech** — Katedra kvantitatívnych metód a ekonomickej informatiky, PEDAS ŽU, Žilina, SR, balint@fpedas.uniza.sk
- Bezegová Ľudmila** — Ústav matematických vied PF UPJŠ, Košice, SR, ludmila.bezegova@student.upjs.sk
- Bukovský Lev** — Ústav matematických vied PF UPJŠ, Košice, SR, lev.bukovsky
- Buša Ján** — Katedra matematiky a teoretickej informatiky FEI TU, Košice, SR, jan.busa@tuke.sk
- Dvorská Ľubomíra** — Gymnázium Šrobárova, Košice, SR, lubomira.dvorska@gmail.com
- Farkašová Zuzana** — Ústav matematických vied PF UPJŠ, Košice, SR, zuzana.farkasova@student.upjs.sk
- Fenovčíková Andrea** — Katedra aplikovanej matematiky Sjf TU, Košice, SR, andrea.fenovcikova@tuke.sk
- Hnatová Jana** — Metodicko-pedagogické centrum RP, Prešov, SR, jana.hnatova@mpc-edu.sk
- Hudák David** — Ústav matematických vied PF UPJŠ, Košice, SR, david.hudak@student.upjs.sk
- Ivančo Jaroslav** — Ústav matematických vied PF UPJŠ, Košice, SR, jaroslav.ivanco@upjs.sk
- Ivanová Veronika** — Ústav matematických vied PF UPJŠ, Košice, SR, veron.ivanova@gmail.com
- Jendroľ Stanislav** — Ústav matematických vied PF UPJŠ, Košice, SR, stanislav.jendrol@upjs.sk

**Rusnačko Rastislav** — Ústav matematických vied PF UPJŠ, Košice, SR,  
rastislav.rusnacko@student.upjs.sk

**Schrötter Štefan** — Katedra matematiky a teoretickej informatiky FEI  
TU, Košice, SR, stefan.schrotter@tuke.sk

**Soták Roman** — Ústav matematických vied PF UPJŠ, Košice, SR,  
roman.sotak@upjs.sk

**Spišiak Ladislav** — Gymnázium Šrobárova, Košice, SR,  
la31.sp@srobarka.sk

**Stas Michal** — Katedra matematiky a teoretickej informatiky FEI TU,  
Košice, SR,  
michal.stas@tuke.sk

**Široczki Pavol** — Ústav matematických vied PF UPJŠ, Košice, SR,  
siroczki@gmail.com

**Škrabuľáková Erika** — Katedra aplikovanej matematiky SjF TU, Košice,  
SR, erika.skrabulakova@tuke.sk

**Šugerek Peter** — Ústav matematických vied PF UPJŠ, Košice, SR,  
peter.sugerek@student.upjs.sk

**Šupina Jaroslav** — Ústav matematických vied PF UPJŠ, Košice, SR,  
jaroslav.supina@student.upjs.sk

**Vojtová Michaela** — Department of Mathematics and Statistics,  
FoS MU, Brno, CzR, misasula@gmail.com

Editori: Ján Buša, Stanislav Jendroľ, Štefan Schrötter

ISBN 978-80-553-0658-2

Sadzba programom pdfT<sub>E</sub>X

Copyright © 2011 Ján Buša, Stanislav Jendroľ, Štefan Schrötter